TOPOLOGY STRUCTURAL OPTIMIZATION USING A HYBRID OF GA AND ESO METHODS

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ABSTRACT

To design a more economical structural form, it is necessary to optimize both the topology and shape of structures. To optimize topology, we propose a hybrid of Genetic Algorithm (GA) and Evolutionary Structural Optimization (ESO). This paper describes the considerations in applying the proposed method to topology structural optimization. Through numerical examples, the proposed method showed better search ability than GA or ESO methods alone. Moreover, this hybrid method makes it possible to design a more economical structural form.

KEY WORDS

Genetic Algorithm, Evolutionary Method, Topology Structural Optimization, Optimization

1 Introduction

To optimize a structural form, it is necessary to optimize topology. Generally, topology structural optimization is a problem of minimizing the volume under various constraints. The most typical method for topology structural optimization is the evolutionary structural optimization (ESO) approach [1], in which a design domain that is not structurally active is considered to be used inefficiently and can be removed by various element rejection criteria (deletion rate). ESO is a method for designing an optimal structural form by deleting unnecessary parts. As it does not require special equipment for the structural calculations, it is an effective method with both extensibility and generality. Various other types of structural topology optimization method have been developed. Among the most well-established of these methods are those based on the homogenization approach [2]. The homogenization method represents the design domain as porous materials with infinite micro-scale cells, and considers the size and angle of the hole of the micro-structure to be a design variable. Moreover, the density method [3] to which the homogenization method can be simplified assumes the material characteristics to be proportional to the involution function of the density. A more recent development is the method based on the cellular automata method, by which the mechanism of growth is expressed [4].

On the other hand, Genetic Algorithms (GA) [5] are optimization methods that simulate the heredity and evolution of living organisms. GAs are utilized for global optimization problems with two or more constraints and allow the optimal solution to be found easily without falling into local solutions. Therefore, GAs are applicable to topology structural optimization, with effects that are equivalent or better than other methods. In application of GAs to topology structural optimization, it is necessary to consider encoding to an object problem, crossover methods for inheritance of parental characteristics, fitness calculations, and constraints for displacement or stress.

In our approach, elements of the structure are encoded by representing one structure as one individual by the chromosome. This approach involves deciding the topology of the structure by allocating the index that shows the presence of each individual element, and application of the structure to topology structural optimization is simple. However, the chromosome length increases in structures with a large number of elements in this approach. Moreover, to find an optimal solution, GAs generally require large numbers of both individuals and generations, and convergence slows.

Therefore, we examined the considerations in applying GAs to topology structural optimization. Moreover, we propose application of the concept of removing unnecessary parts of the ESO method to GAs to improve convergence of a structural form. We then compare with structural forms designed by the proposed method and the ESO method.

2 Evolutionary Structural Optimization

The ESO method proposed by Xie [1] designs the optimal structural form by a simple process deleting unnecessary parts, while repeating normal structural analysis. This method is effective and has both extendibility and generality, because it does not require special equipment for structural optimization calculation.

To determine which parts are unnecessary, in the ESO method, the parts where the stress value and the influence on the whole of the structure is small are deleted. The parts where the structure is analyzed at the generation and the stress value is small are deleted individually based on a given threshold (deletion rate) decided beforehand prior to structural calculation. Therefore, the process of deletion is advanced by the deletion rate for all the processes of evolution, and is irrelevant to the structural form at each stage. Evolution is inefficient when there are many parts that should be deleted, while an economic structure cannot be designed if there are few parts to be deleted. The deletion rate is usually set lower than necessary. Figure 1 shows the case in which unnecessary parts are deleted. As a result of structural analysis, the parts where the stress value is small are deleted only at the deletion rate α in the number of all elements.

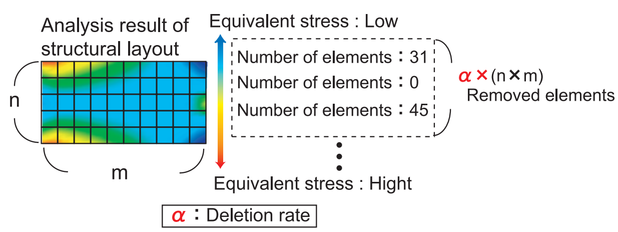


Figure 1. Removed elements of unnecessary parts using ESO method

In this study, to apply GAs to topology structural optimization and for comparison with the ESO method, the structural form designed with the ESO method is assumed to be the optimal layout. Figure 2 shows the process of evolution of a structural layout using the ESO method when the boundary of the structure is fixed on the left side, and the vertical direction concentration load is received at right-center. Figure 2 shows a cantilever problem of receiving a concentrated load, which is a problem faced in many studies. The number of elements of the initial layout is 400, the deletion rate is 0.05, and the structural layout to perform ESO in ten steps is assumed to be the optimal layout. Table 1 shows the final number of elements, maximum displacement, maximum equivalent stress, and decentralization of element equivalent stress on the optimal layout.

3 Structural optimization using GAs and ESO

This study is performed to design a structural layout of a minimum volume under the constraint of maximum displacement, and GAs are applied to topology structural optimization. In this chapter, we describe the procedure for designing a structural layout using GAs and the ESO method.

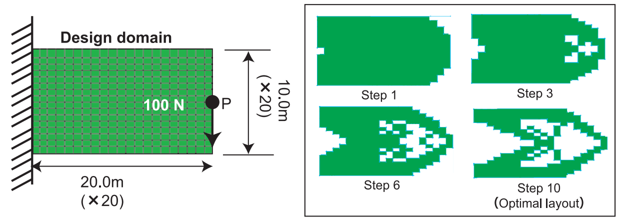


Figure 2. Evolutionary process of a structural layout using the ESO method

Table 1. Results of analysis of the optimal layout

Number of elements	200
Maximum displacement ($\times 10^{-8} m$)	6.474
Maximum equivalent stress ($\times 10^{2} Pa$)	3.491
Decentralization of element equivalent stress ($\times 10^{3}$)	6.876

3.1 Procedure of the proposed method

GAs are optimization methods that simulate the heredity and evolution of living organisms. In GAs, the population of individuals are generated as the initial search points, and individuals are assigned fitness values. Then, the operations (selection, crossover, and mutation) are applied repeatedly to individuals in the population. GAs can find an optimal solution by selection, which selects individuals with high fitness, crossover, which results in inheritance of good characteristics from the parents, and mutation, which maintains diversity by changing parts of the individuals. Moreover, the Distributed Genetic Algorithm (DGA) is a model in which GAs are parallelized. In DGAs, the population is divided into sub-populations (islands), and the genetic operations are performed in each sub-population. Therefore, DGA is also called the Island model. Moreover, DGA includes an operation called migration in which some individuals are transferred to other islands every certain number of generations. The interval of migration and rate of individuals by migration are called the migration interval and migration rate, respectively. DGA has been shown to be able to find better solutions than GAs [6]. In this study, DGA was applied to maintain the diversity of individuals. The procedure of the proposed method is as follows.

1. Initial individuals are generated and initialized.
2. The genetic operators (selection, migration, crossover, mutation) are applied to individuals.
3. The elements of individuals are removed: ESO method.

4. Structural analysis (analysis of the displacement of each node, and stress of each element).
5. Constraints are judged, and the pulling back method is applied to the offspring.
6. Filtering.
7. Offspring are evaluated.
8. The above procedures 2-7 are repeated to the end generations.

These operations are explained in detail in the following sections.

3.2 Coding and fitness

3.2.1 Coding

As mentioned above, in this study, elements of structure are encoded by representing one structure as an individual by the chromosome. Figure 3 shows the coding of determination of the topology of a structural layout by representing the presence of each element by a bit-array (1 bit or 0 bit). Young's modulus E is used as an index representing the presence of elements. In this case, the Young's modulus $E_1(=206GPa)$ is allocated to solid elements where elements are present, and an extremely small Young's modulus $E_0(=103MPa)$ is allocated in void elements where the elements are not present. The topology of the structural layout is represented by the extreme difference between two Young's modulus.

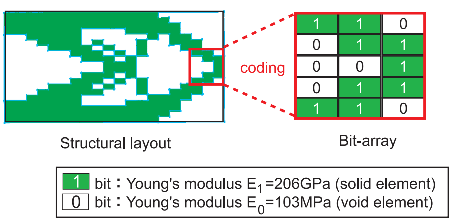


Figure 3. Coding by bit-array representation

3.2.2 Fitness

Under the constraint that the maximum displacement does not exceed the allowable displacement, the fitness is formulated as a problem of minimizing the volume. In the first term of Equation 1 and Equation 2, the total of solid elements possessed by the individual x of chromosome length n is minimized. The fitness increases in the individual with small numbers of solid elements.

In the second term of Equation 1 and Equation 2, the coefficient ζ is multiplied by the allowable displacement ratio of the allowable displacement δ_g, which is the constraint value of the maximum displacement, and the maximum displacement δ_{max} is added to the first term. This is because the maximum displacement is different if the topology is different even if the number of elements is equal. Then, between individuals with an equal number of elements, the fitness of the individual the maximum displacement has margin for the allowable displacement is improved.

In the third term of Equation 2, the decentralized value of element equivalent stress σ_v is scaled with γ, and coefficient ξ is multiplied. Then, γ is $1.0e+3$ to become a scale equal to the second term. This is because the element equivalent stress is different if the topology is different even if the number of elements is equal, similarly to Equation 1. Generally, when stress is concentrated, cracks occur frequently in the structure compared with cases in which the stress is distributed over a wider area. Therefore, cracking becomes les likely in structures with small decentralization of the element equivalent stress. As mentioned above, in Equation 2, the number of elements is considered in the first term, and the allowable displacement ratio is considered in the second term. In addition, the fitness of the individual with a more economical structural can be improved by the decentralization of element equivalent stress considered in the third term. The differences in the design of a structural layout by Equation 1 and Equation 2 are examined by simulation as described in Chapter 4.

$$Minimize : F = \sum_{i=1}^{n} x_i + \zeta \left(\frac{\delta_{max}}{\delta_g} \right) \tag{1}$$

$$x_i \in \{0,1\} \qquad Subject\ \ to : \delta_{max} < \delta_g$$

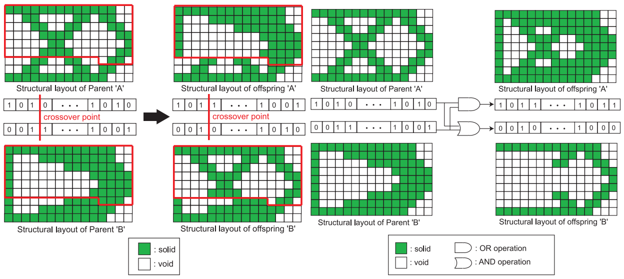
$$Minimize : F = \sum_{i=1}^{n} x_i + \zeta \left(\frac{\delta_{max}}{\delta_g} \right) + \xi \left(\frac{\sigma_v}{\gamma} \right) \tag{2}$$

$$x_i \in \{0,1\} \qquad Subject\ \ to : \delta_{max} < \delta_g$$

3.3 Crossover that considers structural form

When GAs are applied to object problems, it is important to consider an effective crossover method to allow inheritance of good characteristics of the parents by the offspring. When one-point (multipoint) crossover is applied in usual GAs (Genotype) to topology structural optimization, crossover is done without any relation to structural form (Figure 4(a)). As a result, the characteristics of the parents are destroyed, and it is possible that the layout may show a checkerboard density distribution [7].

Therefore, we propose a crossover method that takes structural form into consideration. Figure 4(b) shows the proposed crossover method. Two offspring are generated with the logical OR operation and the logical AND operation of two chromosomes (structural layout of parent 'A' and structural layout of parent 'B'). In the structural layout of offspring 'A', if parent individual 'A' and either 'B' element includes solid elements, the elements become solid elements. In the structural layout of offspring 'B', the elements become solid elements only when both parents include solid elements. One is an individual with the small maximum displacement with large numbers of elements, and another is an individual with large maximum displacement and a small number of elements. The proposed crossover method takes structural form (solid and void elements) into consideration.



(a) One-point crossover (b) Crossover taking structural form into consideration

Figure 4. Comparison of structural layouts according to crossover method

3.4 Using ESO and pulling back method

3.4.1 Improvement of search ability using ESO

Generally, in GAs, the degree of evolution slows so that large numbers of individuals and generations are required to find an optimal solution. Therefore, we propose that the degree of evolution should be increased by deleting unnecessary parts, as in ESO, in the evolutionary process of GA. Moreover, as described in Chapter 2, the ESO method is effective for topology structural optimization. Therefore, a better search ability can be expected by applying the ESO method to the evolutionary process of GA compared with the single GA search.

3.4.2 Pulling back method

In applying the ESO method and the evolutionary process of GA, the pulling back method is applied to offspring that transgress the constraint of allowable displacement.

Figure 5 shows the pulling back method. When offspring 'a' generated from parents 'A' do not fill the constraint and transgress the feasible area, the design variables of parents 'A' are compared to those of offspring 'a'. Then, the elements that are void elements of offspring 'a' and are solid elements of parents 'A' are made into solid elements. As a result, offspring have a larger number of solid elements than the parents, the maximum displacement of offspring becomes small, and the offspring are pulled back to the feasible area. Although the pulling back method results in the number of elements of offspring becoming greater than that of the parents, as mentioned above, because the fitness of an individual is decided by the number of elements, the allowable displacement ratio, and the decentralization of element equivalent stress, offspring can remain in the next generation.

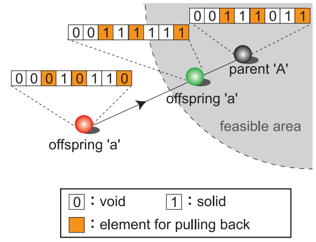


Figure 5. Pulling back method

3.5 Filtering

When the number of elements of structures is of a low-dimension, by the evolutionary process of GA, application of the ESO method, and the pulling back method, a checkerboard-like density distribution may appear frequently in a structural layout [7]. The filtering method, as shown in Figure 6, is indispensable to prevent this density distribution [8]. In the neighboring elements of the object element, if surrounding elements are solid, the object element is made into a solid element. If they are void elements, the object element is made into a void element. Then, the Moore neighborhood is used to decide the neighborhood elements of the object element.

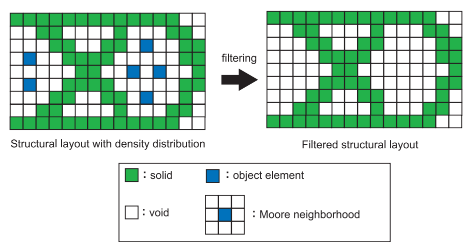


Figure 6. Filtering method using Moore neighborhood

4 Numerical examples

To examine its effectiveness, the proposed method is applied to the cantilever problem of two dimensions. By assuming Step 10 in Figure 2 to be the optimal layout, and using Equations 1 and 2, we examine the design of the structural layout with allowable displacement as a constraint and also when decentralization of element equivalent stress is considered. The methods for examining are GA+ESO to which the concept of the ESO method is applied and GA to which it is not applied.

4.1 Cantilever design optimization

Figure 7 shows the cantilever problem. In Figure 7, the Poisson's ratio υ is 0.3, the number of elements is 400, the number of nodal points is 882, the Young's modulus

$E_1 is 206GPa$ (solid elements), and E_0 is $103MPa$ (void elements). The boundary of the structure is fixed on the left side, and the vertical direction concentration load of $(100N)$ is received at right-center.

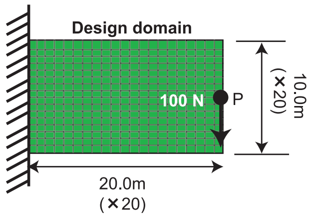


Figure 7. Cantilever problem

Table. 2 shows the parameters of the proposed method. In Table. 2, the deletion rate of GA+ESO is 0.0025, 0.0050, 0.0075, and 0.0100, and it of GA is 0.0. Allowable displacement value is an analysis result of the optimal layout of Table. 1.

Table 2. Parameters of the proposed method

Population size	100
Number of islands	10
Number of elites	1
Chromosome length	400
Migration rate	0.5
Migration interval	5
Mutation rate	0.0025 (1/chromosome length)
Deletion rate	GA+ESO (0.0025C0.0050, 0.0075, 0.0100), GA (0.0)
Allowable displacement value ($\times 10^{-8}m$)	6.474
Coefficient ζ	1.0
Coefficient ξ	1.0
Number of generations	1000

4.2 Design of structural layout with allowable displacement as a constraint

In the constraint of allowable displacement using Equation 1, the results of structural layout and search ability are shown. GA+ESO and ESO methods are also compared.

Figure 8 shows the final structural layouts designed by GA+ESO and GA. Figure 8(d) is similar to the optimal layout in Figure 2. In Figure 8(e), the shape is similar to the optimal layout in Figure 2 although the topology is markedly different.

Figure 9 shows search abilities of GA+ESO and GA. GA+ESO (0.0100) showed greater search ability. On the other hand, that of GA was poorer than that of GA+ESO, which was thought to be because GA does not include the process of deleting unnecessary parts as in the ESO method.

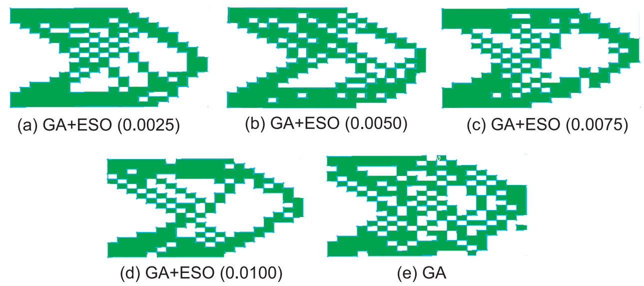


Figure 8. Final structural layout

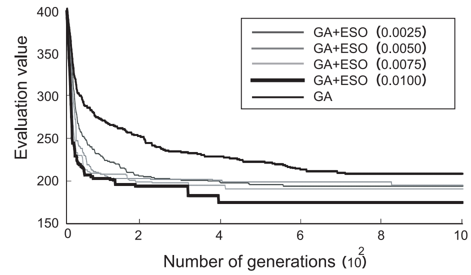


Figure 9. Search ability

Table 3 shows the comparison between GA+ESO (0.0100) and ESO (Optimal layout). GA+ESO was better than ESO with regard to both the number of elements and the maximum displacement. As a result, GA+ESO had better fitness.

Table 3. Comparison between GA+ESO (0.0100) and ESO (Optimal layout)

	GA+ESO	ESO
Number of elements	168	200
Maximum displacement ($\times 10^{-8}m$)	6.455	6.474
Fitness	169.0	201.0

4.3 Design of structural layout when decentralization of element equivalent stress is considered

In the constraint of allowable displacement using Equation 2, which considers decentralization of element equivalent stress, the results of the structural layout and search ability are shown. GA+ESO and ESO methods are also compared.

Figure 10 shows the final structural layouts designed by GA+ESO and GA. With both methods, structural layouts were markedly different from Figure 8 and the optimal layout in Figure 2. Especially, while the optimal layout has an axisymmetric topology around the right-center part, GA in Figure 10(e) showed a topology in which a large number of elements are concentrated on the lower right. This was thought to be because the entire element equivalent stress was distributed by concentrating the number of elements on the lower side.

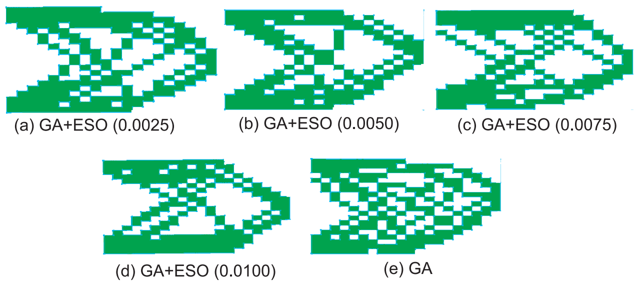


Figure 10. Final structural layout

Figure 11 shows search abilities of GA+ESO and GA. GA+ESO (0.0050) showed better search ability than GA.

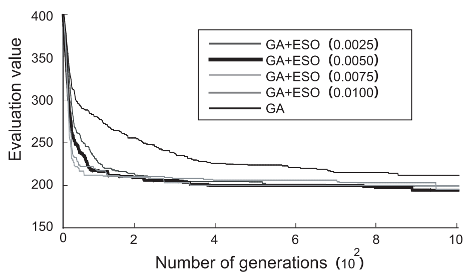


Figure 11. Search ability

Figure 12 shows decentralization of element equivalent stress of GA+ESO and GA. With both methods, decentralization of the element equivalent stress increased from the first stage of the search, and converged. Moreover, that of GA was lower than that of GA+ESO.

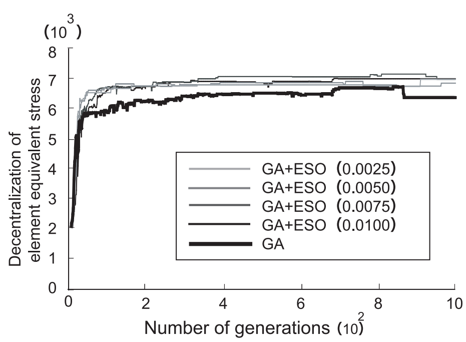


Figure 12. Decentralization of element equivalent stress

Table 4 shows a comparison between GA+ESO (0.0050) and ESO (Optimal layout). GA+ESO was better than ESO with regard to both the number of elements and the maximum displacement. However, ESO was better than GA+ESO with regard to decentralization of element equivalent stress. As a result, GA+ESO had better fitness.

Table 4. Comparison between GA+ESO(0.0050) and ESO (Optimal layout)

	GA+ESO	ESO
Number of elements	183	200
Maximum displacement ($\times 10^{-8} m$)	6.444	6.474
Decentralization of element equivalent stress ($\times 10^3$)	6.935	6.876
Fitness	190.9	207.9

5 Conclutions

In this paper, we proposed a hybrid of GA and ESO methods for topology structural optimization. To apply GA and ESO to topology structural optimization, we examined coding, fitness, effective crossover method, pulling back method, and filtering. The results of the experiments indicated that the proposed method showed improved search ability over the GA or ESO method alone. Moreover, this hybrid method makes it possible to design a more economical structural form than was previously available.

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