DISCUSSION OF SEARCH PHASES OF PROBABILISTIC MODEL-BUILDING GENETIC ALGORITHMS

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ABSTRACT

Recently, the PMBGA has been focused and well developed. The PMBGA uses the statistical information about parents to produce children. It has a high searching ability. In this paper, the difference of the search process between the PMBGA and the canonical GA is discussed. Through the numerical experiments, it is described that the canonical GA has three phases in its search process. On the other hand, the PMBGA has two phases. According to the discussion, it is concluded that the PMBGA can find a good solution with high accuracy and the PMBGA has the high possibility that all the individuals are concentrated on the local minimum.

1. INTRODUCTION

Genetic Algorithms (GAs) are stochastic search algorithms based on the mechanics of the natural selection and natural genetics[1]. The GAs can be applied to several types of optimization problems by encoding design variables to individuals.

Recently, a new type of GA that is called the "Probabilistic Model-Building GA (PMBGA)" has been focused[2]. In the canonical GA, children are generated from the parents and these parents are selected randomly. However, in the PMBGA, the good characteristics of parents are forced to inherit to children using statistical information. Since children must have parents' characteristics, the effective searching is expected. It is reported that the PMBGA has the higher search ability than that of the canonical GA[3, 4]. Because the PMBGA uses the information about parents when children are generated, the search process of the PMBGA may be different from that of the canonical GA.

In this paper, the difference of the search process between the PMBGA and the canonical GA is discussed. At first, the search process of the GA is classified into three phases by the variance of the objective function value. Through the numerical experiments, the way of searching for the solutions in each phase in the canonical GA is discussed. In the same way, the search process of the PMBGA is examined through the numerical examples. According to the comparison of these search processes, the difference of the mechanisms of search of the canonical GA and PBMGA is described.

The paper is organized as follows: The next section represents that the search process of the canonical GA can be classified into three phases by the variance of the objective function values for certain objective problems. Then it mentions elitism and genetic operators (crossover, mutation). Finally, the behavior on the Probabilistic Model-Building GA is discussed through the numerical experiments.

2. SEARCH PHASE OF CANONICAL GA

2.1. Discussion of Search phase of Canonical GA

In this section, before the discussion of the PMBGA, the search phase of the canonical GA is classified and summarized.

Usually in the GA, the initial population is randomly generated at first. Therefore, diverse individuals exist in population on the early phase of the search. Individuals which have the better fitness than previous generation are increased by the iteration of selection. Selection spreads the same chromosome in a population and loses the diversity of population as progress of search. Following numerical results show this behavior. Optimization problems are the minimization of the Rastrigin function (Formula 1) and the Griewank function(Formula 2). The optimal value is 0 when the design variables are 0. Parameters and genetic operators are summarized in Table 1. All the numerical results are the average of 20 trials.

$$Rastrigin = 10n + \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i)\right) \quad (1)$$
$$(x_i \in [-5.12, 5.12], \quad n = 30)$$
$$Griewank = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \left(\cos\left(\frac{x_i}{\sqrt{i}}\right)\right) \quad (2)$$
$$(x_i \in [-512, 512], \quad n = 30)$$

Table 1: Parameters and operators

Population Size	512
Number of Elites	1
Coding	Gray Coding
Chromosome Length (L)	300 (30×10)
Selection Type	Roulette Selection
Crossover Type	1-Point Crossover
Crossover Rate	0.8
Mutation Rate	0.0033334 ($\frac{1}{L}$)

2.1.1. Classification of search phase and evaluation value

Fig. 1 and Fig. 2 show the history of the best and the variance of the function evaluation values respectively. The smaller evaluation value is the better solution which is found by the GA for the minimization problem. When the variance is large, the various evaluation values exist in a population and the diversity is maintained.

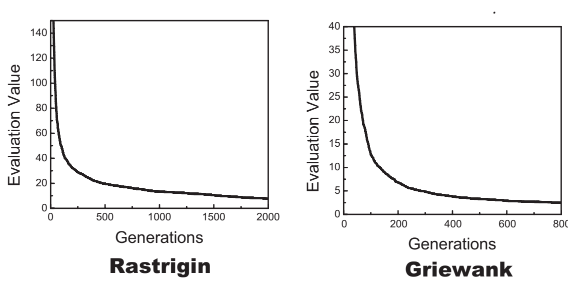

Figure 1: History of best of the function evaluation value

In Fig. 2, the variance keeps on decreasing in the early stage of the search process and starts increasing in a certain generation: about 160 generation for the Rastrigin function and about 80 generation for the Griewank function. The variance becomes steady state after about 600 generation for the Griewank function. In Fig. 1, the best evaluation value converges on the optimal value rapidly in the early stage of the search process. Its convergence becomes slow as the variance starts to increase.

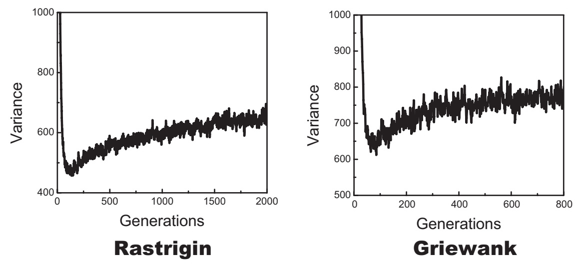

Figure 2: History of variance of the function evaluation value

Therefore, we consider that the search process of the GA can be classified into the following three phases for these two test functions(Fig. 3).

phase 1 The variance of the function evaluation value decreases rapidly. At the same time, the best of the function evaluation value converges on the optimal value rapidly.

phase 2 The variance increases. The improvement of solution is slow.

phase 3 The variance stays a certain value. This may be caused by the stagnation of the evolution of population or the discovery of the optimal solution.

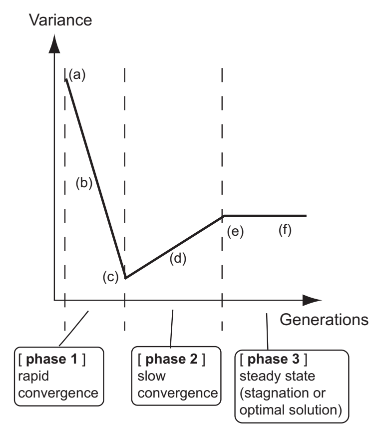

Figure 3: Search process of the GA and the variance of the evaluation value

2.1.2. Distribution of evaluation value on each search phase

This section discusses the reason for the existence of the three phases that are found in Fig. 3in the search process of the GA. Fig. 4 shows the distribution of the function evaluation value for the Griewank function. The left edge of

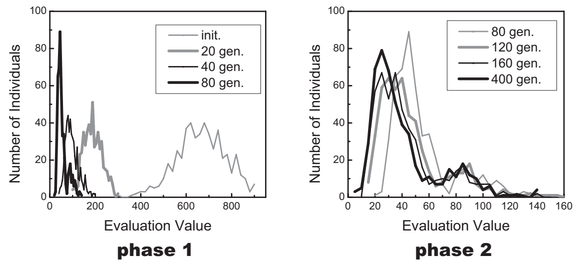

Figure 4: Search process of the GA and the distribution of the function evaluation value. Vertical axis means number of individuals in each range of evaluation value. Left figure is for phase 1 and right one is for phase 2. 'init' means 0 generation.

distribution corresponds to an optimal solution because the optimal value of this function is 0.

In the phase 1, all the individuals are shifting toward the optimal value as search progress. The variance keeps on decreasing during this phase because the difference between the maximum and minimum of evaluation value becomes small.

On the other hand, the fitness values of bad individuals do not improve and only the evaluation values of good individuals keep on improving in the phase 2. The variance increases during this phase since the difference between the maximum and minimum of evaluation value are extending.

2.2. Effectiveness of elitism

Numerical results in Section 2 show that the evaluation values of the bad individuals are not improved and the evaluation values of good individuals keep on improving in the phase 2. Therefore, it may be mentioned that relatively good individuals in each generation play an important role in search in the phase 2. In this section, this aspect is discussed through the numerical experiment.

Four types of experiments to discuss the elitisms are summarized in Table 2. Elitism A increases the number of elites in the phase 2. Elitism B decreases the number of elites in the phase 2. In elitism C and elitism D, the number of elites is constant during the search process. Elitism D preserves $\frac{1}{4}$ of population size as elites.

Table 2: four types of elitism

	number of elites	
	phase 1	phase 2
elitism A	1	64
elitism B	64	1
elitism C	1	1
elitism D	64	64

The parameters not mentioned above and the objective problems are the same as numerical experiments in Section 2. All the results are the average of 20 trials. The boundary between the phase 1 and phase 2 is 160 generation for the Rastrigin function and is 80 generation for the Griewank function, which are the same as Section 2

Fig. 5 shows the history of the best of the evaluation value. The smaller evaluation value is the better solution that is derived by the GA for the minimization problem. Fig. 6 shows the best evaluation value in a certain generation: 320 generation for the Rastrigin and 160 generation for the Griewank. The phase 1 and phase 2 include the same number of generations.

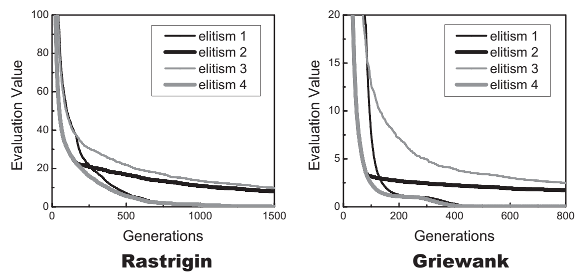

Figure 5: History of best of evaluation value and elitisms

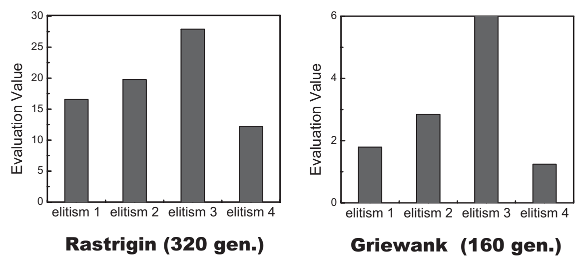

Figure 6: Best of evaluation value (Rastrigin: 320gen. Griewank: 160gen.)

The results presented in these figures indicate that the elitism D is the best and the elitism A is the worst from the point of view of the search performance. Therefore, there is a tendency that the search performance of the GA with multiple elites is higher than that of the GA with one elite.

The elitism A finds the better solution than the elitism B does. This fact indicates that the GA with many elites in the phase 2 finds the better solution than with many elites in the phase 1. Therefore, the strategy that increases the number of elites in the phase 2 is effective for two types of optimization problems.

2.3. Effectiveness of genetic operators

Usually, the similar chromosomes spread in population as the search of the GA progresses. It is reported that crossover plays more important role than mutation when population loses diversity[5]. Therefore, it can be conjectured that mutation is more effective than crossover in the phase 2. This aspect is discussed through the following numerical experiments.

The parameters not mentioned above and the objective problems are the same as those that are used in the numerical experiments in Section 2. All the results are the average of 20 trials. The boundary between the phase 1 and phase 2 is 160 generation for the Rastrigin function and is 80 generation for the Griewank function, which are the same as Section 2

2.3.1. Effectiveness of crossover

In this section, the effectiveness of crossover on each phase is discussed through the numerical experiment. In this section, the following three experiments are prepared for the discussion.

crossover A crossover rate is 0.8.

crossover B crossover rate is 0.8 on phase 1 and is 0.0 on phase 2.

crossover C crossover rate is 0.0.

Only the mutation generates new individuals in crossover C.

Fig. 7 shows the history of the function evaluation value. Since this is the minimizing optimization problem, the smaller value of the solution is the better solution.

The crossover contributes to the search performance for the Rastrigin function because the crossover A finds the better solution than that of the crossover C. The crossover does not work effectively in the phase 2 because the crossover B shows the nearly equal to the search performance of the crossover A. For the Griewank function, the crossover A finds the better solution in the early stage of the search process. However, the solution of the crossover A becomes inferior to the crossover C in the search performance as search progress.

The crossover B and crossover C, where crossover rate is 0.0 in the phase 2, show the higher performance than the crossover A in the late stage of the search process.

Therefore, the crossover is important in the phase 1. On the other hand, the crossover does not work effectively in the phase 2 for these test functions.

2.3.2. mutation

In this section, the effectiveness of the mutation on each phase is discussed through the numerical experiment. In this section, the following three experiments are applied to discuss the effectiveness of the mutation.

mutation A mutation rate is $\frac{1}{L}$ (L : chromosome length).

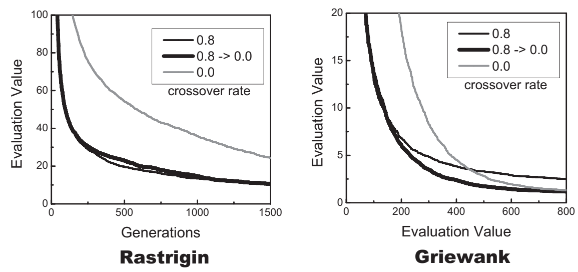

Figure 7: Hisotry of the function evaluation value in each crossover type

mutation B mutation rate is $\frac{1}{L}$ in the phase 1 and is 0.0 in the phase 2.

mutation C mutation rate is 0.0 in the phase 1 and is $\frac{1}{L}$ in the phase 2.

Only the crossover generates new individuals when mutation rate is 0.0.

Fig. 8 shows the history of the function evaluation value. Since this problem is also a minimizing optimization problem, the smaller value of the evaluation is the better. Accuracy of the solution obtained in the mutation A is nearly equal to the one in the mutation C. The mutation does not work effectively and the crossover is more important in the search process in the phase 1. The mutation B finds the better solution than that of the mutation A in the early stage of the phase 2.

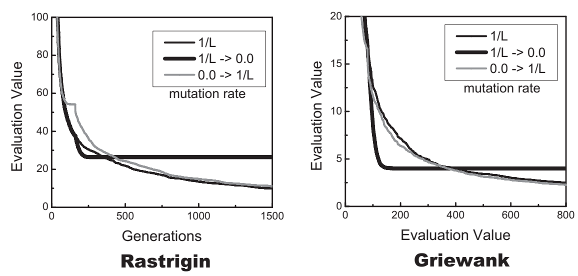

Figure 8: Hisotry of the function evaluation value in each mutation type

3. SEARCH PHASE OF PROBABILISTIC MODEL-BUILDING GA

In this section, it is discussed through the numerical experiment whether search process of the Probabilistic Model-Building GAs(PMBGAs) is different from that of the canonical GA.

The PMBGAs are the algorithms that use a probabilistic model of promising solutions to guide further exploration of the search space[2]. The procedure of the PMBGA used in this section is as follows.

1. A certain ratio of the best individuals are selected from population.
2. The correlation of the design variables correspond to the selected individuals is erased by Principal Component Analysis (PCA).
3. According to the distribution of the design variables, new individuals(offsprings) are generated by normal random numbers.
4. The correlation of the design variables correspond to the offsprings is revised. And the offsprings replace population.
5. In the mutation, a design variable is changed to a certain value in the feasible region by the mutation rate.

The ratio of the number of selected individuals to the population size is 0.1. The mutation rate is $\frac{1}{nVar \times 10}$(nVar : the number of the design variables). The parameters not mentioned above and the objective problems are the same as those that are used in the numerical experiments in Section 2. All the results are the average of 20 trials.

Fig. 9 shows the history of the variance of the function evaluation values. In Fig. 9, the variance keeps on decreasing rapidly in the early stage of the search process and it becomes the steady state. The variance does not start increasing. This is dissimilar to Fig. 2. The fact that offsprings are generated by the probabilistic model instead of the crossover in the PMBGA may make PMBGA's behavior different from the canonical GA. In the canonical GA, only the elite individuals are developing a good point and the rest of individuals keep the diversity in the phase 2. On the other hand, in the PMBGA, it seems that all of individuals are developing a good point in the phase 2. That may be the reason there is no phase where the variance becomes increase. Therefore, it can be said that the PMBGA can find a solution that has high accuracy. At the same time, in the PMBGA, there is a possibility all of individuals are concentrated on the local minimum.

4. CONCLUSION

In this paper, we discuss the search phase of the canonical GA and PMBGA through the numerical experiments.

We classify the search process of the GA into three phases by the variance of the objective function value. In phase 1, all the individuals are shifting to the optimal value as the search progress. The variance keeps on decreasing during this phase because the difference between the maximum and the minimum of evaluation value becomes small. On the other hand, the evaluation values of bad individuals do not improve and the only the evaluation values of good individuals keep on improving in the phase 2. The variance increases during this phase since the difference between the maximum and the minimum of evaluation value is extending.

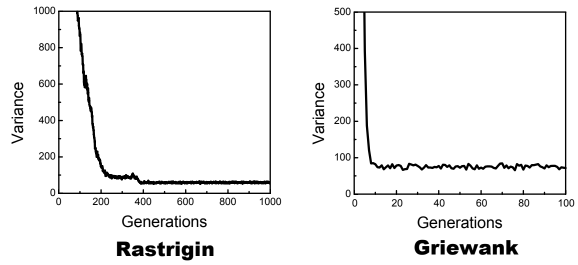

Figure 9: History of variance of the function evaluation value (PMBGA)

In the PMBGA, there are only two phases. They are the phase where the variance of the fitness decreases rapidly and the phase where the variance decreases gradually. That means there is no phase 2 of the canonical GA.

According to these experiments, it can be said that the PMBGA can find a good solution with high accuracy. At the same time, the PMBGA has the high possibility that all the individuals are concentrated on the local minimum.

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A. SOURCE

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