# Adaptive Temperature Schedule Determined by Genetic Algorithm for Parallel Simulated Annealing

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**Abstract- Simulated annealing (SA) is an effective general heuristic method for solving many combinatorial optimization problems. This paper deals with two problems in SA. One is the long computational time of the numerical annealings, and the solution to it is the parallel processing of SA. The other one is the determination of the appropriate temperature schedule in SA, and the solution to it is the introduction of an adaptive mechanism for changing the temperature. The multiple SA processes are performed in multiple processors, and the temperatures in the SA processes are determined by a genetic algorithms. The proposed method is applied to solve many TSPs (Traveling Salesman Problems) and JSPs (Jobshop Scheduling Problems), and it is found that the method is very useful and effective.**

**Key Words : Simulated Annealing, Genetic Algorithm, Adaptive Temperature, Traveling Salesman Problems, Jobshop Scheduling Problems**

## 1 Introduction

There is a strong incentive to parallelize the computation for solving optimization problems since it requires many iterations of analysis. Especially, simulated annealing, which are very effective for solving complicated optimization problems with many optima, requires tremendous computational power. Consequently, parallelization of the method is very important.

It was Kirkpatrick et al. who first proposed simulated annealing, SA, as a method for solving combinatorial optimization problems[1]. It is reported that SA is very useful for several types of combinatorial optimization problems. However, the most remarkable disadvantages are that it needs a lot of time to find the optimum solution and it is very difficult to determine the proper cooling schedule.

Because of the progress of parallel computers, there are several studies on SA using parallel computers[2, 3]. Among these studies, the temperature parallel simulated annealing (TPSA), which was called the time-homogenous parallel annealing[4] before, is one of the algorithms that can overcome the cooling schedule problem, and that can reduce the computation time also.

However, the higher temperatures assigned to some of the processors of a parallel computer can be considered to be too high as the annealing proceeds since the annealing at the higher temperature does not yield the convergence of solutions. Therefore, the effectiveness of multiple processors is somewhat reduced in TPSA.

In order to overcome this problem, we propose a new method for determining the temperature adaptively as the multiple annealings proceed. The temperatures assigned to all the processors of a parallel computer are determined by a genetic algorithm(GA)[5]. The temperatures are dynamically changed to appropriate values during the annealing process.

## 2 Important Temperature Region for TSP

### 2.1 Important Temperature Region

There is an important temperature region in a temperature schedule of SA, where the search is carried out very efficiently. Harry[6] found that a specific constant temperature in SA yields good solutions for TSPs, and Mark[7] obtained the similar results for quadratic assignment problems.

Such specific constant temperatures are called the important temperature regions in SA in this paper, and our proposed method is based on the important temperature region.

### 2.2 Traveling Salesman Problems

The traveling salesman problem (TSP) used in this paper is a problem for finding the minimum distance of a tour of visiting all the finite number of cities and returning to the starting point. The tour distance is expressed as follows[8]:

$$\sum_{i=1}^{N-1} d(v_\pi(i), v_\pi(i+1)) + d(v_\pi(N), v_\pi(1)) \qquad (1)$$

where $v(i)$ is the $i$-th point (city) in a tour $\pi$ , $d(v(i), v(j))$ is the distance between two points, and $d(v(i), v(j)) = d(v(j), v(i))$.

The neighborhood structure used in this paper is the 2-change neighborhood[8], which is the most fundamental one for TSPs.

### 2.3 Confirmation of the Existence of the Important Temperature Region

The important temperature region for each TSP is found by performing many SAs with various constant temperatures and comparing the qualities of the solutions obtained.

The 32 temperatures used are determined by the following procedure, which is commonly used for determining temperatures in TPSA[8].

1) Determine the maximum temperature, where the worst transition is accepted with the probability of 50%. The worst transition is determined by some preliminary experiments.

2) Determine the minimum temperature, where a bad transition is accepted only once for a prescribed annealing steps (20 times the number of cities).

3) Divide geometrically the range between the maximum and minimum temperatures into 32 temperatures.

In order to find the important temperature region, typical six TSPs (Traveling Salesman Problems) from TSPLIB[9] were solved by SA with various constant temperatures.

The one of the experimental results is shown in Fig.1, where the tour distances of kroA200, which is one of the TSPs used, are shown as a function of constant annealing temperature. The values shown are the average of 20 trials. The number of annealing steps are $200N$, $800N$, and $3200N$, where $N$ is the number of cities.

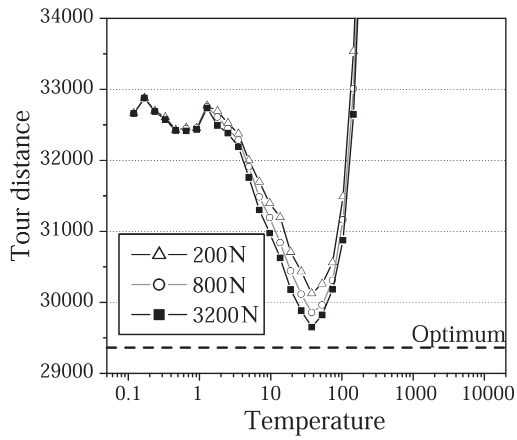


Figure 1: Result of SA with constant temperature for kroA200

From Fig.1, nearly optimum solutions are found between 25 to 50, and this region can be considered as the important temperature region for the kroA200 problem. Similar results are obtained for the other TSPs, and the detail of the experimental results are summarized in Table 1, where $T_{opt} region$ means the important temperature region, and *Optimum* means the obtained tour distance averaged over 20 trials.

Table 1: Important temperatures for several TSPs

| TSP | Optimum | $T_{opt}$ *region* |
|---|---|---|
| eil101 | 629 | 1.1∼2.5 |
| kroA200 | 29368 | 26.8∼52.7 |
| lin318 | 42029 | 19.5∼39.0 |
| pr439 | 107217 | 44.3∼72.3 |
| rat575 | 6773 | 1.7∼3.9 |
| d657 | 48912 | 13.5∼26.8 |

### 2.4 Characteristics of the Transition of the Solution

The transitions of the solution in the important temperature region in SA is investigated in order to find the mechanism for providing good solutions. The histories of the transitions of the solution for eil101 are shown in Fig.2, where the three histories of the tour distance for three temperatures are shown as a function of the annealing steps. One temperature is the important one, one is the maximum temperature, and the other is the minimum temperature.

It can be found that the annealing at the maximum temperature shows a medium fluctuation, and the solution is far from the optimum solution. On the

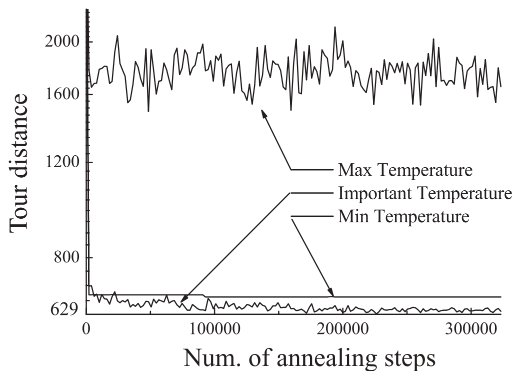


Figure 2: Transitions of solutions at different temperatures (eil101)

other hand, the annealing at the minimum temperature shows relatively good solution, but the fluctuations are very little. Therefore, the possibility of finding the global optimum solution is very low. Compared with these results, the annealing at the important temperature shows a large fluctuation and the shortest distance. Therefore, even the annealing at constant important temperature can provide a very near optimum solution.

The characteristics of the transition of the solution at the important temperature are as follows:

1) Relatively medium fluctuations

2) Relatively good solution

Using these characteristics, a new method for determining the important temperature can be constructed and a new adaptive SA can be devised based on the important temperature.

## 3 Parallel SA with Adaptive Temperature

### 3.1 Concept of PSA/AT(GA)

We found the important temperature region in SA, but such temperature range is problem-dependent and it is difficult to find during the search. Therefore, we consider an adaptive mechanism for determining the important temperature for parallel SA (PSA). This method is called the parallel simulated annealing with adaptive temperature determined by GA (PSA/AT(GA)). Each temperature of PSA is determined by a GA (genetic algorithm). The schematic of the proposed method is shown in Fig.3. It should be noted that the temperature can be evolved since multiple SAs are performed in parallel, and the population of the temperatures can be constructed.

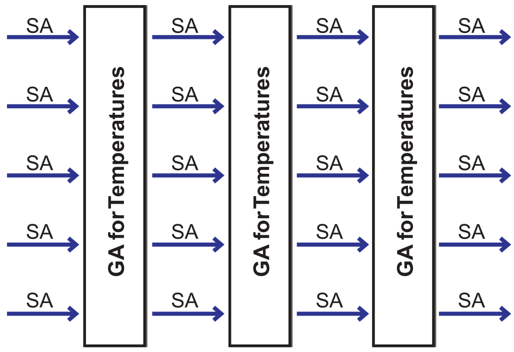


Figure 3: Schematic of PSA/AT(GA)

### 3.2 Algorithm of PSA/AT(GA)

Fig.4 shows the algorithm of PSA/AT(GA), where the initial temperatures are generated with random numbers and multiple SAs starts with these temperatures. After the prescribed annealing steps the temperatures are evolved by using GA operators, and new temperatures are assigned to the multiple SAs, where the annealings are continued.

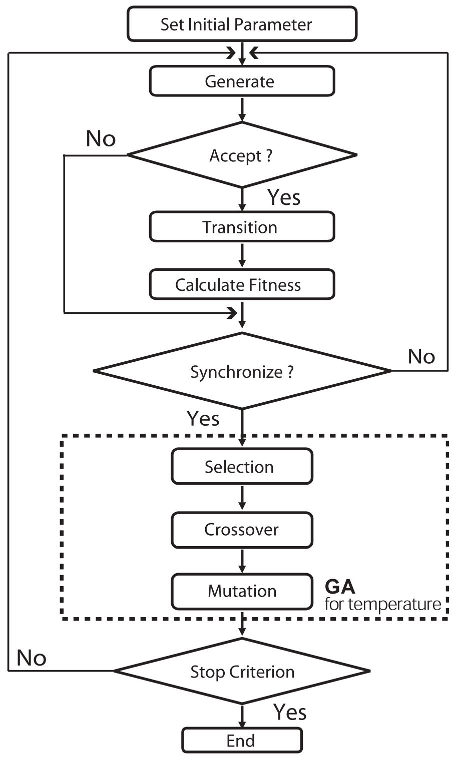


Figure 4: Algorithm of PSA/AT(GA)

The features of the method is as follows.

1) Generation, Acceptance, Transition

The generation of a new solution, the acceptance criterion, and the transition of the solution are the same as conventional SAs.

2) Temperature is determined by GA

The synchronization of all the processes is done with a certain period, and the temperatures are determined by a GA based on the fitness value calculated from the energy history of the solutions.

The temperatures of the multiple SA processes are changed by crossover, mutation, and selection. That is, the temperature that gives good solutions survives. Thus, all the temperatures are expected to be converged to the important temperature region.

### 3.3 Fitness value

The fitness value for the selection of better temperatures are defined based on the characteristics mentioned in section 2.4, as follows.

$$Fitness = \sum_{k=1}^{L} (\overline{E} - E_k) \tag{2}$$

where $E_k$ is the energy at $k$-th annealing steps in an interval, and $\overline{E}$ is the average value of all energies, and $L$ is the tempreature change interval. That is, the fitness is the summation of the differential energy values lowered below the averaged energies of all SA processes performed in parallel. With this fitness value, the temperature with the characteristics of the transitions at the important temperature can be selected as a good temperature.

### 3.4 Coding of temperature

The temperature in SA is a very important parameter and the performance of a SA is heavily dependent on an appropriate temperature schedule[10]. In practice, the exponential cooling schedule is widely used. Therefore, the temperature in PSA/AT(GA) is expressed by Eq.(3).

$$Temperature = 10^X \tag{3}$$

where $X$ is a binary coded number. Therefore, the range of temperature can be very wide. The crossover used is the one-point crossover.

For a continuous desing variable, the real-valued encoding is often used, but the appropriate design of the crossover and mutation is not easy in such case. Therefore, we adopt the simple binary encoding.

## 4 Numerical Experiments

### 4.1 Outline of experiments

To verify the validity and the effectiveness of the proposed method, PSA/AT(GA) and TPSA are applied to solve six TSPs.

The parameters used are shown in Table 2. The number of temperatures, which is the same as the number of SA processes, and the total anealing steps are the same as in Ref.[8]. The maximum and minimum temperatures are determined so as to include the important temperature region in this range surely.

Table 2: Parameters used for PSA/AT(GA)

| | |
|---|---|
| Num of SA processes | 32 |
| Temperature change interval | 5N |
| Total steps | 5N×160 |
| bit length | 10 |
| Selection method | Roulette |
| Crossover rate | 0.3 |
| Mutation rate | 0.01 |

N : number of cities

For the TPSA, the temperatures are determined by the conventional method mentioned in section 2.3, and the interval for exchange solutions are the same as the temperature change interval shown in Table 2.

### 4.2 Parallel computer used

The parallel computer used is a PC cluster (Cambria cluster system) with 256 processor elements, and 32 nodes are used for the experiments. The detail of the computer system is shown in Table 3.

Table 3: Detail of the PC cluster used

| | |
|---|---|
| CPU | Pentium3 800MHz(256CPU) |
| Memory | 256MB×256 |
| Network | FastEthernet |
| OS | Debian GNU/Linux 2.4 |
| Communication | mpich |

### 4.3 Experimental results

The typical histories of the tour distance for PSA/AT(GA) and TPSA are shown in Fig.5, where the values are the average of 20 trials. Compared with PSA/AT(GA) and TPSA, TPSA shows a good convergence at the beginning, but PSA/AT(GA) shows a better performance at the later stage. The same results are obtained for the other problems. Therefore, it can be concluded that PSA/AT(GA) has a better performance in searching the global optimum than TPSA.

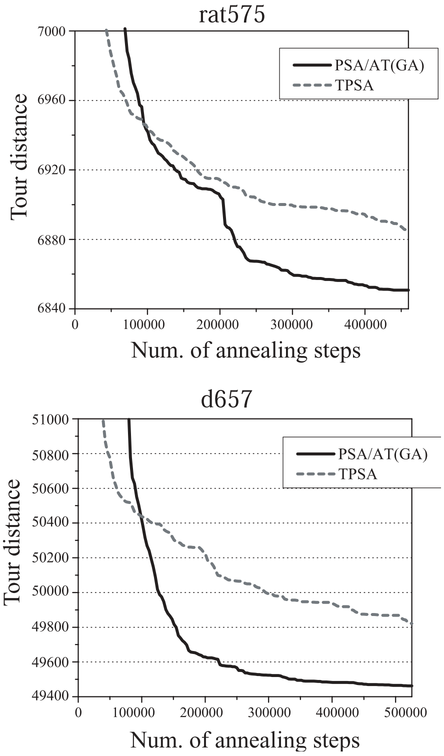


Figure 5: Energy histories for PSA/AT(GA) and TPSA

The typical histories of the temperature in PSA/AT(GA) are shown in Fig.6, where all the 32 temperatures of the 32 annealing processes are shown. The initial temperatures are determined randomly so that they spread over the very large range of temperature, but they converge quickly to a certain value, and fluctuate around it. This fluctuation is generated by the crossover and mutation in the GA operation, and the converged temperature is the important temperature for each problem.

From this result, all the temperatures in PSA/AT(GA) are converged to a narrow range, and this range corresponds to the important temperature region shown in Table 1. On the other hand, the temperatures vary in a complicated manner in TPSA, and there is no indication that the temperatures are gathered in the important temperature region. The reason PSA/AT(GA) shows high performance for searching optimum solution is that the temperatures in PSA are determined adaptively so that they are included in the important temperature region for a given problem.

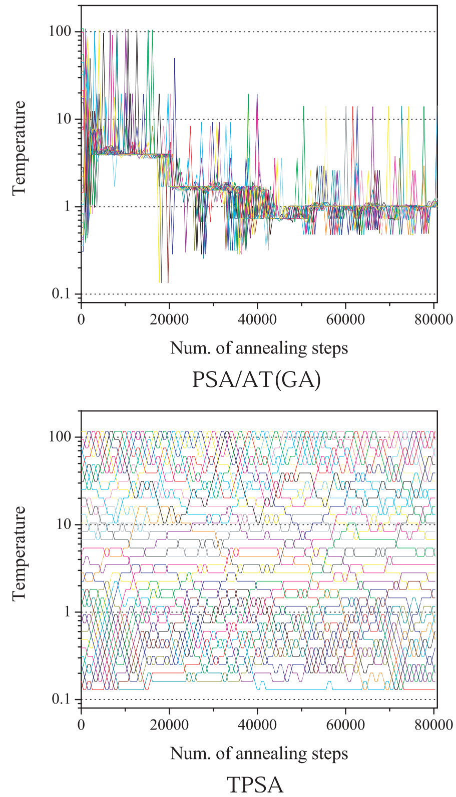


Figure 6: Temperature schedules of PSA/AT(GA) and TPSA for eil101

The best histories of the temperatures in one annealing process that yields the best solution among 32 annealing processes run in parallel are shown in Fig.7, and the difference in the history of the temperature between PSA/AT(GA) and TPSA can be clearly seen. In TPSA relatively low temperatures tend to yield the best solutions, and the contribution of the annealing processes with higher temperatures is found to be little.

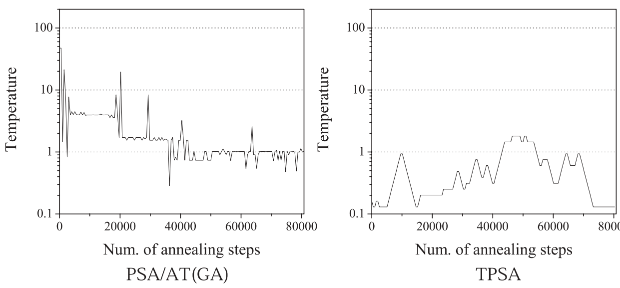


Figure 7: An example of the best temperature schedule for eil101

The comparison of the search performance for

PSA/AT(GA) and TPSA is shown in Fig.8, where the error ratio is the ratio of difference between the exact distance and the obtained distance divided by the exact distance. From this figure, it is concluded that the proposed method shows very high performance in finding the optimum. Consequently, PSA/AT(GA) is found to be very effective and useful method.

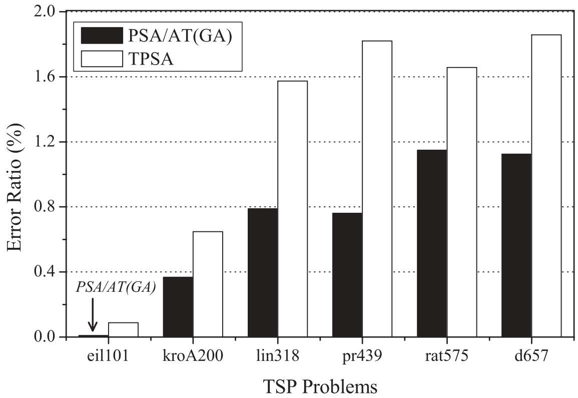


Figure 8: Comparisons of error ratios in TSP

Table 4 shows the comparison of the calculation time for PSA/AT(GA), TPSA and Sequential SA(SSA) with 3 problems. The speedup for PSA/AT(GA) and TPSA is also shown in the parenthesis based on the time for SSA.

Table 4: Comparison of PSA/AT(GA),TPSA and SSA in the execution time

| | the execution time[sec] (Speedup) | | |
|---|---|---|---|
| Problems | PSA/AT(GA) | TPSA | SSA |
| eil101 | 21.8( 9.3) | 10.1(20.0) | 202(1.0) |
| kroA200 | 54.9(19.0) | 41.5(25.1) | 1043(1.0) |
| lin318 | 89.8(22.5) | 76.0(26.6) | 2025(1.0) |

It is found that the calculation time for PSA/AT(GA) is a little longer than TPSA, but the speedup for PSA/AT(GA) and TPSA increases as the problem size becomes longer. Therefore, the proposed method shows high parallel efficiency.

## 5 Application of PSA/AT(GA) to JSPs

The proposed method is applied to solve JSPs (Jobshop Scheduling Problems). The generation of the next solution is done by the critical block (CB) neighborhood[11], and the feasible solution is obtained by modifying the solution with the GT method[12]. The numerical experiments are carried out with six typical JSPs.

At first, the important temperature region is obtained by the similar experiments mentioned at section 2.3, and the result is shown in Table 5. Thus, the important temperature region is recognized, and it has not been conducted so far.

Table 5: Important temperature region for typical JSPs

| JSP | Optimum | $T_{opt}$ *region* |
|---|---|---|
| FT10 | 930 | 5.8∼14.2 |
| FT20 | 1165 | 3.1∼9.7 |
| ORB1 | 1059 | 7.5∼14.2 |
| ORB3 | 1005 | 7.5∼16.0 |
| LA21 | 1046 | 3.5∼12.5 |
| LA40 | 1222 | 2.7∼12.5 |

PSA/AT(GA) and TPSA are applied to solve these six JSPs, and the search performances are compared. The parameters used in the experiments are shown in Table 6, and the maximum and the minimum temperatures are determined similarly as mentioned in section 2.3.

Table 6: Parameters used for PSA/AT(GA)

| | |
|---|---|
| Num of SA processes | 32 |
| Temperature change interval | 256 |
| Total steps | 320000 |
| bit length | 10 |
| Selection method | Tournament |
| Tournament size | 2 |
| Crossover rate | 0.1 |
| Mutation rate | 0.01 |

Examples of the best temperature schedules which yield the best solutions in PSA/AT(GA) and TPSA are shown in Fig.9, and the similar result is obtained as Fig.7. It can be recognized that the converged temperature in PSA/AT(GA) is the same as the important temperature region shown in Table 5, and the proposed adaptive mechanism works very well in JSPs.

The comparison of the search performance is shown in Fig.10, and it can be found that the PSA/AT(GA) has very high searching ability.

Consequently, the proposed mechanism for determining the appropriate temperatures by GA is found to be very effective, and PSA/AT(GA) can be considered to be a useful parallel SA method.

It should be noted that the temperature adaptation mechanism adopted here can be realized with parallel SA since the criterion for selecting "good" temperatures can be established from the relative value and the relative fluctuation of the energies of the multiple

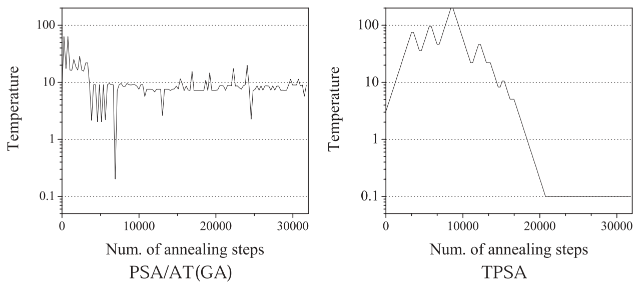


Figure 9: An example of the best temperature schedule for FT10

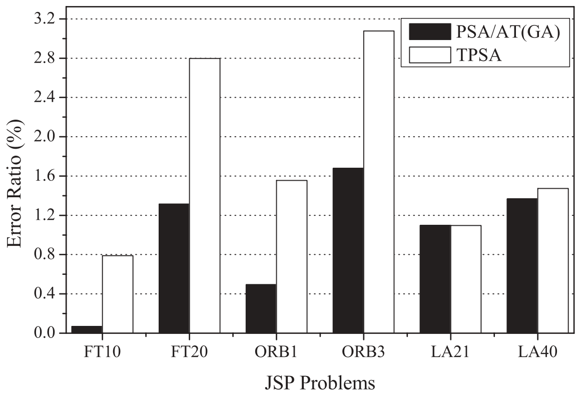


Figure 10: Comparisons of error ratios in JSP

SA processes. From this standpoint of view, parallelization of SA will give another new insight to optimization research field as well as speedup.

## 6 Conclusion

A new parallel simulated annealing method with adaptive temperature mechanism is proposed here. The conclusions are as follows.

1) It is not easy to determine an appropriate temperature schedule for a discrete optimization problem in simulated annealing (SA), but the SA with a specific constant temperature, which is called the important temperature here, can yield very good solutions.

2) The behavior of the transitions of solutions is investigated, and the characteristics of the behavior is found for the solutions at the important temperatures.

3) It is found that the temperatures of parallel SA processes can be optimized using GA and the above characteristics.

4) A new parallel SA with the above temperature adaptation mechanism is proposed, and the effectiveness and the usefulness of the proposed method are shown clearly for the Traveling Salesman Problems and the Jobshop Scheduling Problems. This method is called PSA/AT(GA) and the method is very easy to use since we do not have to determine the temperature schedule, and it gives very good solutions as well.

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