SPEA2+: Improving the Performance of the Strength Pareto Evolutionary Algorithm 2

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Abstract. Multi-objective optimization methods are essential to resolve real-world problems as most involve several types of objects. Several multi-objective genetic algorithms have been proposed. Among them, SPEA2 and NSGA-II are the most successful. In the present study, two new mechanisms were added to SPEA2 to improve its searching ability a more effective crossover mechanism and an archive mechanism to maintain diversity of the solutions in the objective and variable spaces. The new SPEA2 with these two mechanisms was named SPEA2+. To clarify the characteristics and effectiveness of the proposed method, SPEA2+ was applied to several test functions. In the comparison of SPEA2+ with SPEA2 and NSGA-II, SPEA2+ showed good results and the effects of the new mechanism were clarified. From these results, it was concluded that SPEA2+ is a good algorithm for multi-objective optimization problems.

1 Introduction

Shaffer's Vector Evaluated Genetic Algorithm (VEGA[1]) spawned several attempts to apply evolutionary computation to multi-objective optimization problems(MOPs), and several algorithms have been proposed. Among them, SPEA2[2] developed by Zitzler and NSGA-II [3] developed by Deb have been reported to perform well and contain methods useful for multi-objective genetic algorithms. However, in these proposed algorithms, the crossover mechanism, which is an operator in genetic algorithms, has not yet been explored. Many multi-objective genetic algorithms have operations to maintain diversity in the objective space, but diversity in the variable space has not yet been considered.

In this paper, a new algorithm, SPEA2+, is presented. SPEA2+ attempts to improve the problem space exploration abilities of SPEA2 by adding a more effective crossover mechanism and an algorithm to maintain diversity in the two object and variable spaces.

In this paper, SPEA2+ is compared to SPEA2 and NSGA-II to discuss the feasibility of the proposed algorithm.

2   SPEA2+

Many algorithms have been proposed in recent years, and SPEA2[2] proposed by Zitzler has been reported to perform well in searching. SPEA2 contains the important operations such as archiving of individuals with good fitness, density estimation, and fitness assignment, and is able to obtain a population with both "precision" and "diversity".

However, there has been insufficient discussion concerning effective crossover, one of the major operators in GA, in SPEA2. Most multi-objective GAs include operations to maintain a wide diversity of individuals in the objective space, but do not consider the population distribution in the design variable space.

In this paper, SPEA2+ is proposed as a different model to SPEA2 that includes more effetive crossover and a method to obtain diverse solutions in the objective and variable spaces. SPEA2+ adds the following operations to SPEA2:
1) Neighborhood crossover, which crosses over individuals close to each other in objective space.
2) Mating selection, which reflects all archived good individuals in the search.
3) Applying archive to allow holding of diverse solutions in the objective space and variable space.
These operations are explained in the following section.

2.1   Neighborhood crossover

In multi-objective GAs, effective crossover often cannot be performed, as the searching directions of each parent individual are very different from one another. Therefore, we propose neighborhood crossover, which performs crossover with individuals neighboring each other in objective space. In neighborhood crossover, individuals that match in the search direction are crossed over to generate offspring that are similar to the parent. Watanabe reported that the neighborhood crossover mechanism is effective in multi-objective GA[8].

Neighborhood crossover is performed as follows:
Step 1: Sort the population with one of the function values.The function value used in the sort is altered each generation.
Step 2: Neighborhood shuffle is performed for the sorted population.
Step 3: Select ith and $i+1$th items as parents and crossover is performed.

In neighborhood crossover, individuals that are next to each other within the population sorted based on arbitrary function values are defined as neighboring individuals.

To avoid crossing over with the same individuals, the neighborhood shuffling operation is applied after sorting ; neighborhood shuffling counterchanges individuals in the randomized range, which is less than 10% of the population size.

2.2   Mating selection

The operation to create a population by selecting individuals from the archive is called mating selection. Binary tournament selection is used in SPEA2 as a method for mating selection[4]. Binary tournament selection obtains two individuals from the archive and selects the individual with the higher fitness. By this operation, SPEA2 can obtain a population with individuals with high precision solutions.

However, searching with SPEA2 results in an increase in non-dominated individuals within the archive, and in most cases all individuals become non-dominated individuals in the later stages of the search. Thus, use of binary tournament selection to generate the population sacrifices diversity of non-dominated individuals.

Therefore, in the proposed method, as the mating selection method, all of the archive is copied to the population used in the search. This copy operation maintain the diversity of the population to allow for a more global search.

2.3   Archive truncation

The archive truncation method[2] is used to reduce the number of non-dominated individuals when there are more non-dominated individuals than the size of the archive. Two individuals closest to each other in Euclid distance in the objective space within the non-dominated solution are chosen. The distances between second-closest individuals and the chosen individuals are evaluated, and the individual that is closer to its second-closest individual is reduced. By this operation, the archive will hold a more diverse solution in the objective space.

Most multi-objective GAs consider diversity in objective space but not in the variable space. In MOPs, in the final stages of the search, it is necessary to select a good solution from the non-dominated individuals, where the variable values forming the solutions become important. Therefore, if comparable objective function values can be achieved using different design variables, having the diversity of design variables in the non-dominated solution within the variable space is effective.

In the present study, a method with two archives holding diverse solutions in the objective and variable spaces was used. In the operation, non-dominated solutions in each generation are copied to both archives, and in each archive, archive truncation is performed based on both the objective and variable space Euclid distance. The archive will thus hold a diverse range of solutions in both spaces.

2.4   SPEA2+ Algorithm

The algorithm flow of SPEA2+ is as follows:
Input N (archive size)
T (maximum number of generations)
Step 1: Initial population P_0 is generated. A_0^O and A_0^V are the empty archives.

Generation is $t = 0$.

Step 2: Fitness values of all individuals in P_t, A_t^O, A_t^V are calculated with fitness assignment method[2].

Step 3: All non-dominated individuals in P_t, A_t^O, and A_t^V are copied to A_{t+1}^O A_{t+1}^V. If the number of individuals of A_{t+1}^O and A_{t+1}^V have exceeded N, archive truncation in objective space is applied to the individuals in A_{t+1}^O, and archive truncation in variable space is A_{t+1}^V to reduce the number of individuals. If the number of individuals of A_{t+1}^O or A_{t+1}^V is less than N, individuals with good fitness from P_t A_t^O A_t^V are used to fill A_{t+1}^O A_{t+1}^V.

Step 4: Terminate the search if $t \geq T$ or other termination conditions are met.

Step 5: P_{t+1} is generated by copying A_{t+1}^O. The neighborhood crossover and mutation operations are performed. Return to step 2 with $t = t + 1$.

3   Numerical experimentation

To clarify the effects of neighborhood crossover and copy operations, the results of SPEA2_NC and SPEA2_copy, which are SPEA2 with the operations built in, were examined. In addition, the searching effectiveness of SPEA2+ will be discussed. For diversity of the variable space, the results of the variable space archive are compared with SPEA2. To visualize the variable space distribution, 3-variable KUR was used as the test problem. By comparison with NSGA-II[3], which has also been reported to perform well alongside SPEA2, the effectiveness of the proposed method SPEA2+ is discussed.

3.1   Target problem

Several different test functions with different characteristics were used. All of the functions used were minimization problem with 2 objectives. In this experiment, ZDT4[6] reported by Zitzler and Deb, KUR[5] reported by Kursawe, and F_{dis}[7] reported by Deb were used. The formulae of each problem are presented in Table 3.1. However, as stated above, to see the diversity of the variable space distribution, 3-variable F_{dis} was used.

Table4.1   Test problem

Problem	n	Variable bounds	Objective functions
ZDT4	10	$x_1 \in [0,1]$	min $f_1 = x_1$
		$x_i \in [-5,5]$	min $f_2 = g(x)[1 - \left(\frac{f_1}{g}\right)^{0.5}]$
			$g = 1 + 10(N-1) + \sum_{i=2}^{N}(x_i^2 - 10\cos(4\pi x_i))$
F_{dis}	100	$x_i \in [0,1]$	min $f_1 = x_1$
			min $f_2 = g(x)[1 + 10\frac{\sum_{i=2}^{N} x_i}{N-1}]$
			$g = 1 - \left(\frac{f_1}{g}\right)^{0.25} - \frac{f_1}{g}\sin(10\pi f_1)$
KUR	100	$x_i \in [-5,5]$	min $f_1 = \sum_{i=1}^{N-1}(-10\exp(-0.2\sqrt{(x_i^2 + x_{i+1}^2)}))$
			min $f_2 = \sum_{i=1}^{N}(|x_i|^{0.8} + 5\sin(x_i)^3)$

3.2   Comparison method

In this experiment, the precision of the obtained population was evaluated from the ratio of non-dominated individuals as described below. The width of the population was compared with the maximum, minimum, and mean values for each axes of the target function. In this experiment, 30 runs were performed for all target problems. In each run, the ratio of non-dominated individuals, maximum, minimum, and mean values were obtained, and used as the result.The ratio of non-dominated individuals is explained below.

Ratio of non-dominated individuals

Ratio of Non-dominated Individuals (RNI) is a method for evaluation by comparing the dominance of two populations obtained by two different algorithms. In RNI, the populations obtained from the two algorithms, S_1 and S_2 are combined to make a union set S_U. Obtain the set of non-dominated individuals S_P from S_U. The number of individuals contained in S_P from each algorithm is used to obtain the ratio, and the value is used as the result of evaluation. When the value is closer to the maximum value of 100%, the algorithm can be said to have obtained a better population.

3.3   GA Parameters

In this experiment, the population number was set as 100 in all problems. The number of generations for ZDT4 was set to 500. For KUR and F_{dis}, the number of generations was set to 250. The bit-coding method is used for representation of individuals, and the number of bits per variable was set to 20, as used in other studies[2][3]. For mutation, bit-flipping was used, and one-point crossover was used for crossover. The crossover rate was 1.0, and mutation rate was 1/bit-length.

3.4   Performance comparison results

ZDT4

The results for ZDT4 as target problem are shown in Fig.1. The distribution graph shown in Fig.1(a) is the result of collecting all solutions for 30 runs, and the pie chart shows the RNI value compared with SPEA2. Graph (b) shows the means for maximum, minimum, and mean values for each objective function axes.

ZDT4 is a multimodal problem in $f_2(x)$, and the main problem is how to escape from a local optimal value to the Pareto-optimal solution. Fig.1(a) indicates that SPEA2 falls to the local optimal value several times in the 30 runs, and the proposed methods of SPEA2 with neighborhood crossover and copy operations reached closer to the Pareto-optimal solution, although they still fell in the local optimal solution. From Fig.1(b), it can be seen that the mean value for SPEA2+ is the smallest, indicating that on average it is performing a good search.
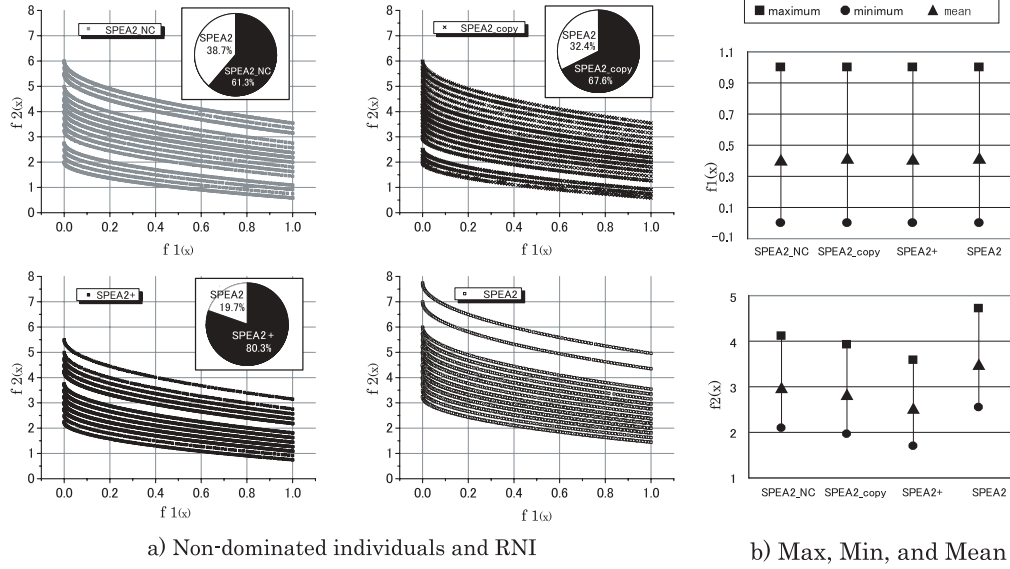
a) Non-dominated individuals and RNI

b) Max, Min, and Mean

Fig. 1. Result ZDT4

F_dis

Fig.2 shows the results using F_{dis} as the target function. The method of presenting the diagram is the same as described for Fig.1.

This is an example of a problem where the Pareto-optimal solution is non-continuous. The number of design variables was 100, and thus the problem is more difficult than ZDT4. As shown in Fig.2(b), the differences in maximum, minimum, and mean values for each target function value axes were not markedly different between the algorithms. However, the SPEA2+ distribution chart in Fig.2(a) shows that the search precision deviated less between each run, and RNI showed that SPEA2+ was better than SPEA2.

KUR

The results obtained with KUR as the target problem are shown in Fig.3. The method of display is the same as described for Fig.1.

This problem has interdependency between neighboring variables on $f_1(x)$ and has multimodal characteristics on $f_2(x)$. From Fig.3(a), it can be seen that SPEA2_NC and SPEA2+ with neighborhood crossover achieved a more diverse population. In addition, SPEA2_copy, which included the copy operation in mating selection, obtained better solutions more often than SPEA2, which used tournament selection.
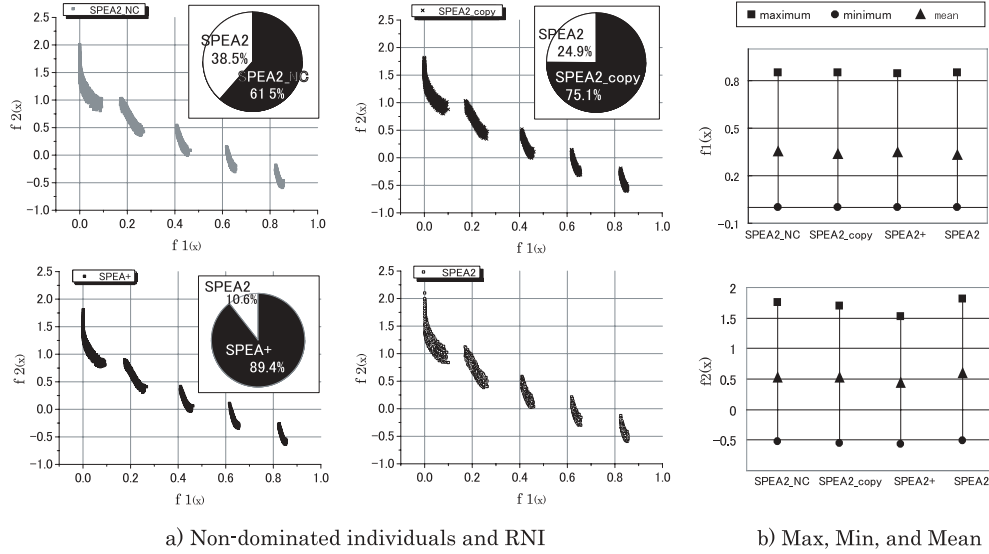
a) Non-dominated individuals and RNI   b) Max, Min, and Mean

Fig. 2. Result F_{dis}

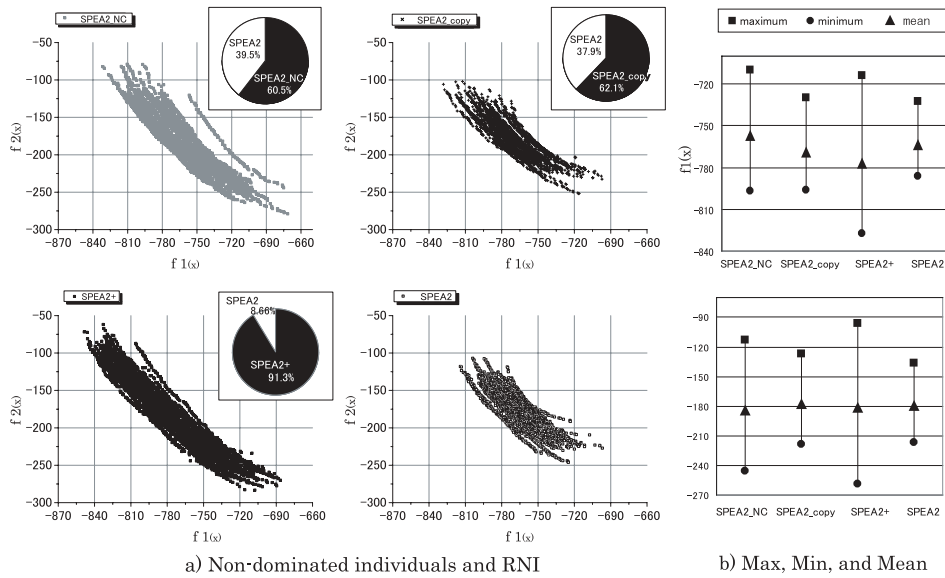a) Non-dominated individuals and RNI   b) Max, Min, and Mean

Fig. 3. Result KUR

3.5   Comparison with NSGA-II

Comparison of proposed SPEA2+ and NSGA-II was performed for each target function. RNI, maximum, minimal, and mean values are shown in Fig.4.

Fig.4 indicates that both algorithms show comparable results in target problem F_{dis}, and the difference in search ability between SPEA2+ and NSGA-II was smaller than that in comparison with SPEA2. This was probably due because congestion is considered in binary tournament selection in NSGA-II mating selection. In tournament selection in NSGA-II, when the fitness values of two individuals being compared are equal, the individual with lower congestion is selected. Therefore, it can generate a more uniformly distributed population than SPEA2+. This focus on population diversity is common with SPEA2+, and in SPEA2+ diversity is maintained by copying all individuals. SPEA2+, which utilizes neighborhood crossover, obtained a wider solution than NSGA-II in the KUR problem, demonstrating the effectiveness of neighborhood crossover in the search.
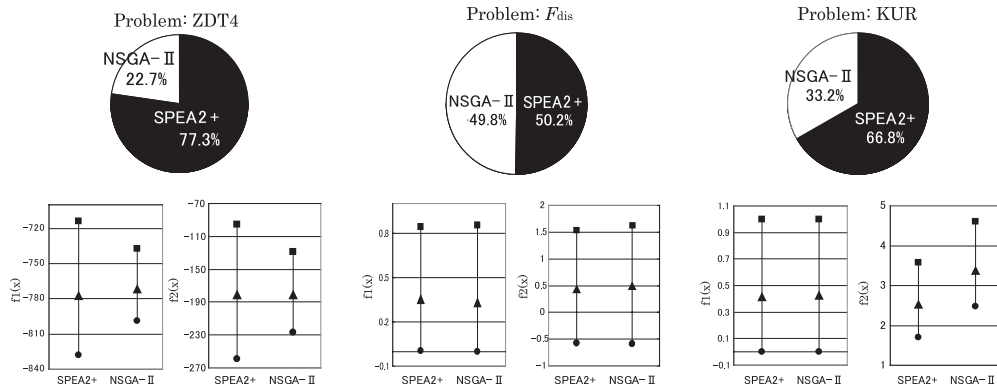

Fig. 4. Comparison of SPEA2+ with NSGA-II

3.6   Comparison by design variable space

The results of the algorithms applied to the three-variable KUR problem are shown in Fig.5. The distribution charts on the top plot the objective archive on the objective space, and the charts on the bottom plot the variable archive on the variable space. The pie-charts show the RNI values in comparison with SPEA2.

This problem is a three-variable problem, and is relatively simple. In each method, the precision of the solution in the target function field resulted in a similar value. On the other hand, the chart on the bottom of Fig.5 shows that SPEA2_twoArchive (which introduced two archives to SPEA2) and SPEA2+ obtained a wider variety of individuals in the variable space. This was caused by having two archives to maintain diversity in both objective space and variable space.
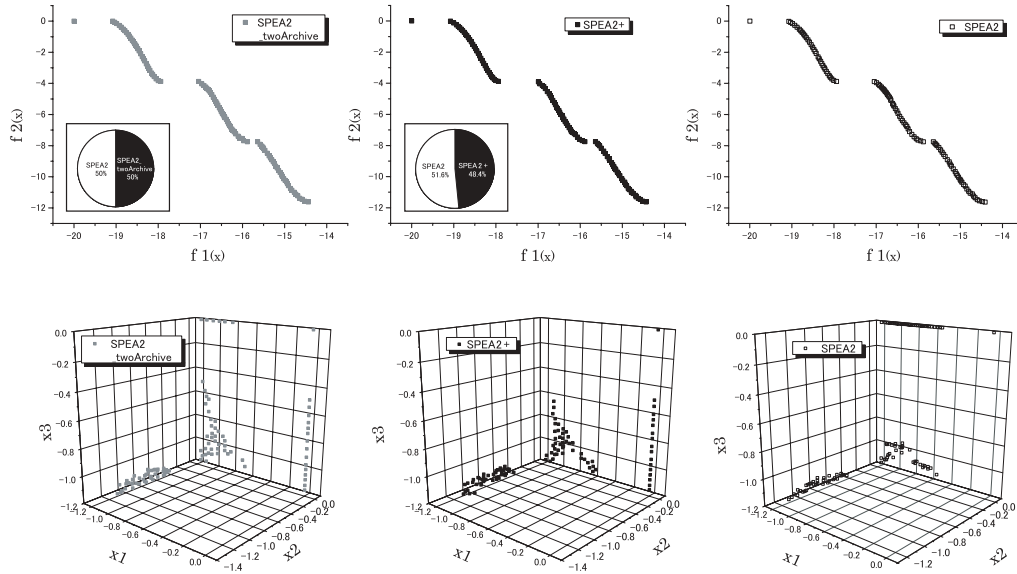
Fig. 5. Distributions of individuals in objective space and variable space

3.7   Discussion of results

In this experiment, three test problems were used for comparison. To clarify the effects of neighborhood crossover and the copy operation proposed in this paper, the results of SPEA2_NC and SPEA2_copy, which are SPEA2 with the respective additions, are shown, and the results of SPEA2+ were compared with SPEA2 and NSGA-II.

The results of the experiments verified the improvement of searching precision after adding copy operation to mating selection. This was considered due to the avoidance of local optimal solutions by generating populations with a wide variety of individuals. On the other hand, when neighborhood crossover was performed, the results showed improved diversity of individuals along with precision . This was considered due to the generation of offspring close to the parents by neighborhood crossover, which maintains a wider range of individuals.

In comparing the diversity of the solutions in the variable space, after introducing two archives, it was possible to obtain a wider variety of individuals in the variable space without affecting the searching ability. This was due to the use of only the objective archive at mating selection for the population. Therefore, individuals in the variable archive do not affect usual searches.

4   Conclusions

In this paper, SPEA2+, which is an improved SPEA2 algorithm, was presented. SPEA2+ is based on SPEA2 with neighborhood crossover to perform better crossover, as well as a copy operation for mating and two archives for maintenance of a wider variety of variable space and objective space.

The experiments yielded the following points:

· By performing neighborhood crossover, population diversity can be obtained.

· By performing copy operation, solutions with better precision can be obtained.

· By using two archives, it is possible to obtain a wider variety of individuals in the variable space without affecting the search ability.

SPEA2+, which included the above operations, mostly showed better results than SPEA2 or NSGA-II. These observations suggest that SPEA2+ is an effective algorithm.

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