Voronoi Model-Building Genetic Algorithm

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***Abstract*— This paper proposes the Voronoi Model-Building Genetic Algorithm (VMBGA), which is one of real-coded GAs. In the VMBGA, a voronoi model is constructed using with voronoi diagrams. Because of this mechanism, the distribution of offspring can adapt to the landscape of the objective function by changing the voronoi model. Through the some standard test functions, the effectiveness of the VMBGA is examined. It is clarified that the VMBGA has higher searching ability than the UNDX-m, which is one of the typical real-coded GAs. Additionally, the distribution of the offspring is also discussed.**

I. Introduction

Crossover design is very important to perform an efficient search in Genetic Algorithms (GAs)[1][2]. In particular, in real-coded GAs, offspring are often generated based on a parent distribution in design space from a small number of parents extracted from a population[3][4][5][6][7]. In addition, in real-coded Probabilistic Model-Building GAs (PMBGAs)[8][9], a probabilistic model is created using the statistical information of individuals who have higher evaluation values, and offspring are generated. The common feature here is the generation of offspring following the average, variance and covariance of the parents. Therefore, as for the offspring generated from a certain parent group, the landscape of the objective function is generally not taken into consideration, and depending on the parent distribution, offspring are not concentrated in regions with a satisfactory evaluation value. In the case of real-coded GA, offspring are often generated using a normal distribution and uniform distribution, but with problems where many local optima exist or problems where local optima are discrete, it is difficult to concentrate offspring for plural local optima simultaneously for a particular generation alternation. Therefore, for such a problem, a large population size is taken, generation alternation is repeated many times, and searches are performed while maintaining the diversity of the population. However, in such an approach, the essential problem in crossover cannot be solved, and many offspring are generated in regions with poor evaluation values as a result. Due to this, offspring not only inherit the distribution of the parents, but also be generated in accordance with the landscape of the objective function.

Therefore, in this paper, the Voronoi Model-Building Genetic Algorithm (VMBGA) which generates offspring by creating a voronoi model using a voronoi diagram is proposed. In the proposed method, the number of voronoi regions is changed according to the landscape of the objective function, and the voronoi model is created by finely dividing parts with a satisfactory evaluation value and roughly dividing parts with a poor evaluation value. By generating offspring with a uniform probability in each region, many individuals are concentrated in parts with a satisfactory evaluation value.

In this paper, the performance of the proposed method was verified by making a comparison with the Multi-Parental Unimodal Normal Distribution Crossover (UNDX-m)[6] which is one of the typical real-coded GAs proposed by Kita et al. The effectiveness of the proposed method was also discussed from the distribution of the generated offspring.

II. Voronoi Model

In this paper, the Voronoi Model-Building Genetic Algorithm (VMBGA) is proposed. In the VMBGA, a voronoi model for generating offspring is created using a voronoi diagram. First, in this section, an outline of the voronoi diagram will be given, and the method of constructing the voronoi model will then be described.

A. Voronoi Diagram

The voronoi diagram[10] decides how to divide the space between the region of each point and its boundary as in Equation 1. In Equation 1, $P = \{\vec{p}_1, \vec{p}_2, \vec{p}_3, ..., \vec{p}_n\}$ is given in a two-dimensional space and $d(\vec{p}_i, \vec{p}_j)$ expresses the Euclidean distance of $\vec{p}_i$ and $\vec{p}_j$.

$$V(\vec{p}_i) = \{\vec{p}|\vec{p} \in \mathrm{R}^2, d(\vec{p}, \vec{p}_i) < d(\vec{p}, \vec{p}_j), j \neq i\} \qquad (1)$$

Fig. 1 shows an example of a voronoi diagram. Each point in the point set P which generates the voronoi diagram is called a voronoi generator, the region divided by certain voronoi generators is known as a voronoi region, and the region $V(\vec{p}_i)$ divided by the points $\vec{p}_i$ shows that, at any arbitrary point in the region, $\vec{p}_i$ is an element which is a grouping of the closest points. The edge of plural voronoi regions is known as a voronoi edge, and the intersection of three or more voronoi edges is known as a voronoi vertex.

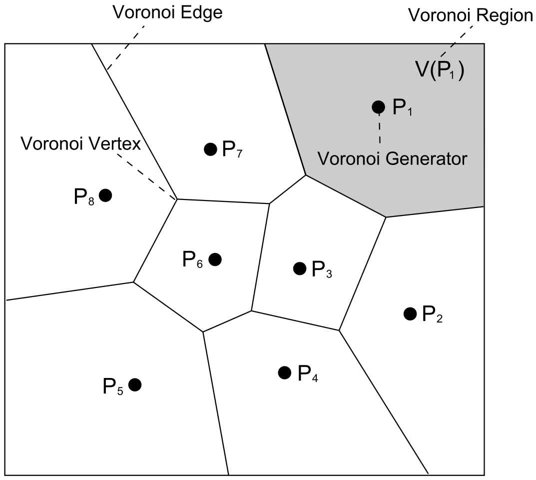

Fig. 1. Voronoi diagram.

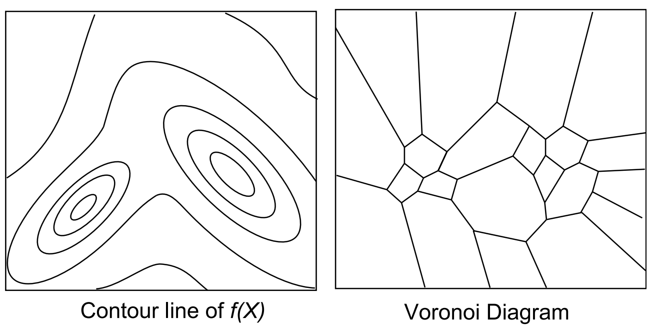

Fig. 2. Contour line of objective function and voronoi diagram.

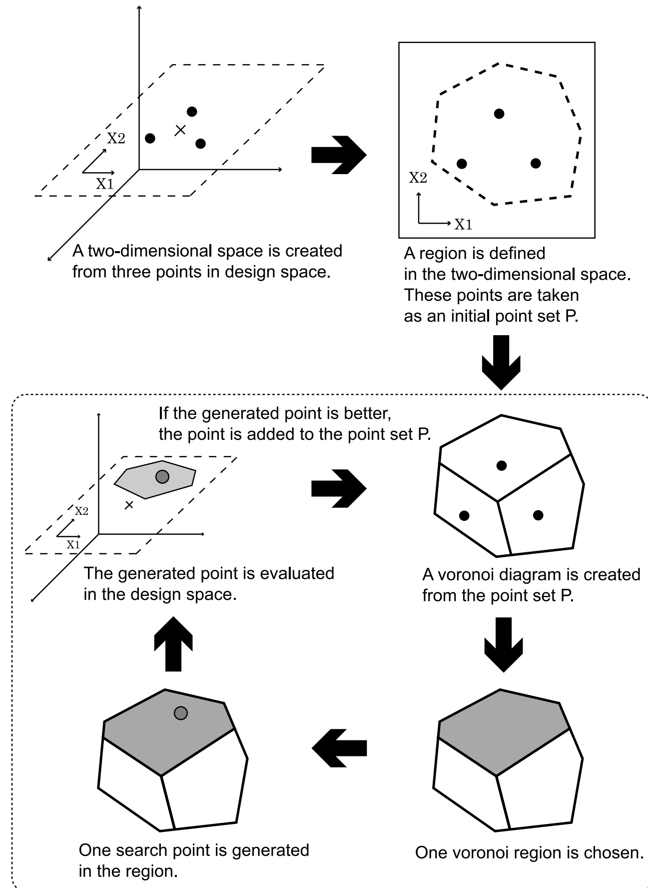

Fig. 3. How to construct the voronoi space.

B. How to Construct the Voronoi Model

In the VMBGA, a two-dimensional space is created from information about parents extracted from a population, and a model for generating offspring is created using a voronoi diagram. Fig. 2 shows an outline of the voronoi diagram which is created. The purpose of the model created by the VMBGA is to change the number of voronoi regions according to the landscape of the objective function. In other words, parts with a satisfactory evaluation value are finely divided, and parts with a poor evaluation value are roughly divided. Hence, if the voronoi generators forming each voronoi region are regarded as offspring, many offspring are generated in the parts with a satisfactory evaluation value, and offspring can be generated in the whole region which created the voronoi diagram without concentrating them in the parts with a satisfactory evaluation value.

As shown in Fig. 3, the following procedure is used for creating N offspring using a voronoi diagram.

1) Three parents are extracted from the population at random, and a two-dimensional space is created in design space.
2) A region is defined for creating a voronoi diagram in the two-dimensional space.
3) Three extracted parents are taken as an initial point set P in the two-dimensional space.
4) A voronoi diagram is generated from the point set P.
5) From the generated voronoi diagram, one voronoi region is chosen at random and one search point is generated in the region using a uniform distribution.
6) The generated search point is evaluated in the design space. When the fitness is higher than a predefined reference point, the generated search point is added to the point set P.
7) When the number of elements in the point set P is less than N+3, it returns to 4. Also, when the number of elements in the point set P is N+3, N search points are taken as offspring except for three parents in the design space.

When N offspring are generated using the voronoi diagram, as there are points which are not added to the point set P in Step 6, N or more evaluations are required. Therefore, how the region which generates offspring is defined, or how reference point is set, leads to efficient generation of offspring.

C. Region for generating Offspring

The VMBGA defines a region for creating a voronoi diagram from three parents extracted from a population as described in Section II-B. This region shows a region for generating offspring in a two-dimensional space. Therefore, the region which generates offspring can be changed in various ways by defining this region. The discussion of the definition of this region will be left for future discussion, but in this paper, for convenience, this region is defined as follows.

First, a center of gravity of parents extracted from a population is found. Next, the distance between the center of gravity and each of the extracted parents is found, and the maximum distance is set to d. Then, based on Equation 2, a radius r is found, and a circular region centered on the center of gravity is defined. This equation is determined referring to the Multi-Parental Unimodal Normal Distribution Crossover (UNDX-m)[6], which is a typical real-coded GA proposed by Kita et al. Moreover, as the region for generating offspring in the primary search component of UNDX-m (m=2) are all satisfied when $amp = 3$, in this paper, amp is taken as 3.

$$r = d/\sqrt{2} * amp, \quad amp = 3 \tag{2}$$

D. Reference Point

When a model is built for offspring generation using a voronoi diagram, a value used as a reference for determining whether a generated individual is added to a point set P based on the fitness of the individual, is required. When the model is built, the setting of this reference point is the most important element. In other words, by setting this reference point appropriately, the number of voronoi regions can be appropriately adjusted according to the landscape of the objective function. If this value is set low, most of the generated individuals are added to the point set P, which is synonymous with searching the region which created the voronoi diagram at random. When this value is set high, most of the generated individuals are not added to the point set P, and the number of evaluations for generating a fixed number of offspring greatly increases. Although the setting of the reference point which can build the model most efficiently will be left for future discussion, this paper defines this value as the average fitness or the worst fitness of the point set P. When the average fitness is used, the reference point becomes higher as more individuals with a high fitness are added to the point set. In other words, the reference point is set with a relatively low value in the initial stage of model construction, and with a comparatively high value at the end.

III. Voronoi Model-Building Genetic Algorithm

A. Overview of the Voronoi Model-Building Genetic Algorithm

The VMBGA proposed by this paper generally indicates a real-coded GA using the voronoi model described in Section II. In VMBGA, it is assumed that the voronoi model is used for part of the determination of coordinate system values used for offspring generation in the crossover method of existing real-coded GA. For example, in BLX-α[3] which is a crossover method of typical real-coded GA, two coordinate system values selected at random can be determined using a voronoi model. Moreover, in the Distributed Probabilistic Model-Building Genetic Algorithm (DPMBGA)[11] proposed by the authors, some of the coordinate system value rotated using principal component analysis can be similarly determined as BLX-α. This paper focuses particularly on UNDX-m which is a crossover method for typical real-coded GA, wherein part of the UNDX-m crossover method is replaced by a determination of values using a voronoi model.

B. Multi-Parental Unimodal Normal Distribution Crossover using Voronoi Model

One well-known crossover method for a real-coded GA is the Unimodal Normal Distribution Crossover (UNDX)[4], [5] proposed by Ono. The Multi-Parental Unimodal Normal Distribution Crossover (UNDX-m)[6] is a technique which extends UNDX. It is considered that these techniques efficiently inherit the character of the parents used for crossover, and efficiently generate offspring. In UNDX-m, m+2 parents are chosen first from a population at random. Then the following algorithm is used for generating offspring:
Let the parental vectors be $\boldsymbol{x^1}, ..., \boldsymbol{x^{m+1}}$, the center of parental vectors be $\boldsymbol{p} = \sum_{i=1}^{m+1} \boldsymbol{x^i}/(m+1)$ and the difference vectors between $\boldsymbol{x^i}$ and $\boldsymbol{p}$ be $\boldsymbol{d^i} = \boldsymbol{x^i} - \boldsymbol{p}(i = 1, ..., m+2)$. Let D be the size of component of $\boldsymbol{d^{m+2}}$ that is orthogonal to $\boldsymbol{d^1}, ..., \boldsymbol{d^m}$. Let $\boldsymbol{e^1}, ..., \boldsymbol{e^{n-m}}$ be an orthonormal bases orthogonal to the subspace spanned by $\boldsymbol{d^1}, ..., \boldsymbol{d^m}$ (n expresses the number of dimensions). Then the offspring $\boldsymbol{x^U}$ is generated in the form

$$\boldsymbol{x^U} = \boldsymbol{p} + \sum_{i=1}^{m} w_i \boldsymbol{d^i} + D \sum_{i=1}^{n-m} v_i \boldsymbol{e^i}, \tag{3}$$

where w_i and v_i are random variables that follow normal distribution $N(0, (1/\sqrt{m})^2)$ and $N(0, (0.35/\sqrt{n-m})^2)$.

In Equation 3, the first two terms are called a primary search component and the third term is called a secondary search component. In the algorithm of UNDX-m, the primary search component formed from m+1 parents can be regarded as a m-dimensional space. Therefore, in this paper, attention is focused on UNDX-2 (m=2), and a voronoi model is used for determination of the value of the primary search component. As m=2, the primary search component is a two-dimensional space. For a generation alternative model, a model which extends the Minimal Generation Gap(MGG)[12] as UNDX-m to plural parents was used.

IV. Numerical Example

In this experiment, to verify the performance of the VM-BGA, UNDX-2 is compared with UNDX-2 using the voronoi model described in Section III-B. The performance difference also is discussed from a comparison of the offspring distribution generated by the two techniques.

A. Target Problems

The test functions in this paper are five functions: Rastrigin function, Schwefel function, Rosenbrock function, Griewank function and Rotated Rastrigin function shown below. The Rotated Rastrigin function is obtained by rotating design variables by $\pi/6$ and applying the Rastrigin function. The Rastrigin function and Schwefel function are multipeak functions which do not have correlation between design variables.

The Rosenbrock function is a single-peak function which has correlation between design variables. The Griewank function and Rotated Rastrigin function are multipeak functions which have correlation between design variables. In this experiment, to verify the performance of VMBGA, a two-dimensional and a four-dimensional problem were used. In the case of the two-dimensional problem, in UNDX-2, only the value of the primary search component is determined and there is no secondary search component. Hence, the performance difference of offspring generation between the voronoi model and a normal distribution can be clearly verified.

$$F_{Rastrigin} = 10n + \sum_{i=1}^{n}\left(x_i^2 - 10\cos(2\pi x_i)\right) \quad (-5.12 \leq x_i < 5.12) \tag{4}$$

$$F_{Schwefel} = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{|x_i|}\right) - C \quad (C : optimum., -512 \leq x_i < 512) \tag{5}$$

$$F_{Rosenbrock} = \sum_{i=2}^{n}\left(100(x_1 - x_i^2)^2 + (1 - x_i)^2\right) \quad (-2.048 \leq x_i < 2.048) \tag{6}$$

$$F_{Griewank} = 1 + \sum_{i=1}^{n}\frac{x_i^2}{4000} - \prod_{i=1}^{n}\left(\cos(\frac{x_i}{\sqrt{i}})\right) \quad (-512 \leq x_i < 512) \tag{7}$$

B. Comparison of the UNDX-2 using Voronoi Model with the UNDX-2

In this experiment, the performance difference of UNDX-2 and UNDX-2 using a voronoi model is compared by numerical simulation. In the numerical simulation, we prepare the following two models of UNDX-2 using the voronoi model.

- Model 1 : The average fitness of the point set P is used for the reference point.
- Model 2 : The worst fitness of the point set P is used for the reference point.

The comparison is made from the number of optima discovered in 20 trials, and the average number of evaluations when the optima are discovered. When the function evaluation value is less than 10^{-8}, it is considered that the optimum has been reached. The search terminating condition was considered as the case where all individuals are concentrated within the limits of 10^{-7} of the width of the design space, when the number of evaluations exceeded $2.0 * 10^6$. Moreover, the number of offspring generated for every generation alternation is verified together with variation of search performance due to number of offspring using 200 and 500. The population size was taken as 15 for a two-dimensional problem, and 40 for a four-dimensional problem. Table I, Table II and Table III show the number of optima discovered for the two-dimensional and four-dimensional problems. Fig. 4 shows the average number of evaluations when the optima are discovered.

TABLE I

NUMBER OF TRIALS WHEN THE OPTIMUM IS FOUND (UNDX-2)

	2 dimensions		4 dimensions	
Number of children	200	500	200	500
Rastrigin	17	20	17	19
Schwefel	16	17	12	13
Rosenbrock	20	20	20	20
Griewank	13	16	4	7
Rotated Rastrigion	19	20	19	19

TABLE II

NUMBER OF TRIALS WHEN THE OPTIMUM IS FOUND (MODEL 1)

	2 dimensions		4 dimensions	
Number of children	200	500	200	500
Rastrigin	20	20	16	15
Schwefel	20	20	13	17
Rosenbrock	20	20	20	20
Griewank	16	20	14	12
Rotated Rastrigion	20	20	18	19

In Table I, Table II and Table III, UNDX-2 using the voronoi model is better than or the same as UNDX-2. In UNDX-2 using the voronoi model, the model using the worst fitness of the point set P for the reference point is better than or the same as the model using the average fitness. This shows that it is more effective to use a voronoi model rather than a normal distribution to determine the value of the primary search component of UNDX-2. With the Schwefel function for which local optima exist near the end of the design space, and the Griewank function for which an infinite number of local optima exist, it is seen that generation of offspring by a normal distribution easily degenerates into local optima, and an effective search is difficult. However, an effective search can still be performed in this type of problem by generating offspring using a voronoi model. On the other hand, when the number of generated offspring was increased, the optimum search performance was improved in generation of offspring by a normal distribution, and in generation of offspring using the voronoi model, the optimum search performance decreased depending on the problem. This will be discussed later.

Fig. 4 describes that more evaluations are required to discover optima for UNDX-2 using a voronoi model in all the functions. This is due to the fact that many evaluations are required to build the voronoi model. Construction of the voronoi model for an efficient search is thus a future problem.

C. Distribution of the generated individuals

To verify the optimum search performance difference of UNDX-2 using the voronoi model and UNDX-2, Fig. 5 shows the distribution of offspring generated from the same three parents in both techniques. The target problems are a two-dimensional Rastrigin function, Schwefel function and Rosenbrock function, and the number of generated offspring is 1000. Fig. 6 shows the distribution of offspring generated from

TABLE III
NUMBER OF TRIALS WHEN THE OPTIMUM IS FOUND (MODEL 2)

	2 dimensions		4 dimensions	
Number of children	200	500	200	500
Rastrigin	20	20	19	20
Schwefel	20	20	18	17
Rosenbrock	20	20	20	20
Griewank	20	20	9	14
Rotated Rastrigion	20	20	19	18

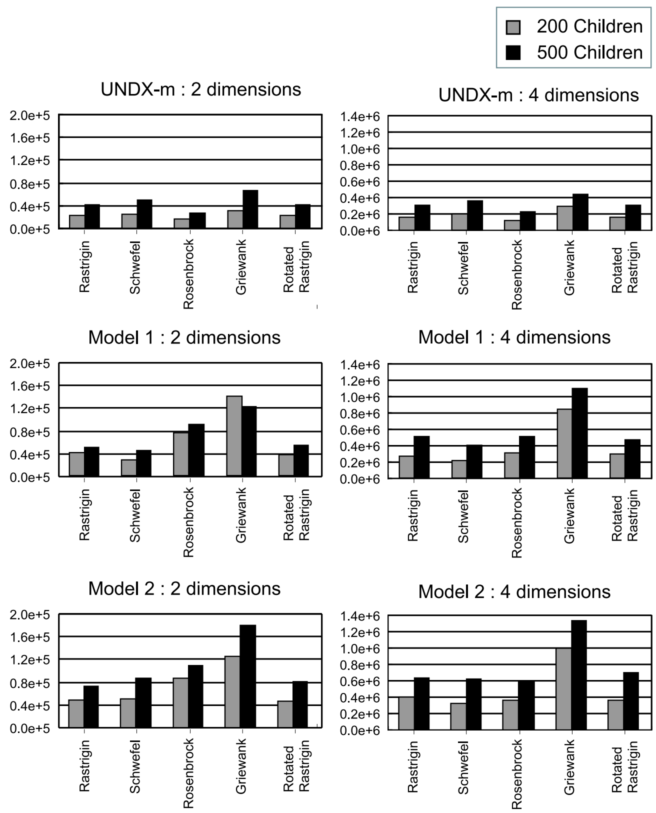

Fig. 4. Average number of evaluations when the optimum is found.

different parents in the models of UNDX-2 using the voronoi model. The target problem is a two-dimensional Schwefel function. Contour lines for each function are shown on the left-hand side of Fig. 5 and Fig. 6, and the positions of the parents are displayed on the contour lines. In UNDX-2 using the voronoi model, all points generated during model construction are also displayed.

Fig. 5 shows that, in offspring generation using a normal distribution, the same offspring are generated from the same parent regardless of the landscape of the objective function. On the other hand, in offspring generation using the voronoi model, offspring are generated mainly in regions having a low function evaluation value, and the distribution of offspring changes in accordance with the landscape of the objective function. In particular, for the Rastrigin function, offspring are generated mainly for plural local optima. This is impossible when individuals are generated using a normal distribution, and from this, it is seen that the proposed offspring generation using a voronoi model is an effective technique.

On the other hand, Fig. 6 shows that offspring generated from two different parents are concentrated in different local optima. In particular, this tendency is strongly found in the results from the model using the average fitness for the reference point. This suggests many search points were generated into local optima discovered in the initial stage of voronoi model construction. Therefore, it is necessary to correct the method of setting a reference point used in constructing the voronoi model. In other words, it appears that when the average fitness of the point set P which creates the voronoi diagram is taken as a reference point, search points generated in regions other than the local optima are difficult to add to the point set P. On the other hand, in regions near the local optima, local search progresses by finely dividing regions, and many search points are added to the point set P. Therefore, it may be considered that search points are no longer generated in any regions other than the local optima discovered in the initial stage. This is thought to be the reason why an increase in the number of offspring generated in the numerical simulation of Section IV-B does not lead to an improvement in search performance. In other words, even if the number of generated offspring is increased, search points are concentrated in specific local optima, and various offspring cannot be generated.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, the Voronoi Model-Building Genetic Algorithm (VMBGA) was proposed. VMBGA was developed by considering the landscape of the objective function and focusing on the effective crossover. In the proposed technique, a voronoi model for generating offspring was created using a voronoi diagram. In this process, the distribution of offspring is changed by altering the voronoi model in accordance with the landscape of the objective function.

Using plural test functions, the effectiveness of the VMBGA was shown as a result of comparison with UNDX-m which is a typical real-coded GA. On the other hand, by comparing the number of evaluations required to discover optima, it was found that an efficient voronoi model must be built. It was also found that the number of offspring generated in accordance with the landscape of a function in the VMBGA can be changed from the distribution of generated offspring. However, the offspring may also be concentrated in specific local optima, and the setting of the reference point in constructing the voronoi model is thus a future problem. Additionally, it is also a future work that the VMBGA is applied to a higher-dimensional problem by using the Extrapolation-Directed Crossover (EDX)[13].

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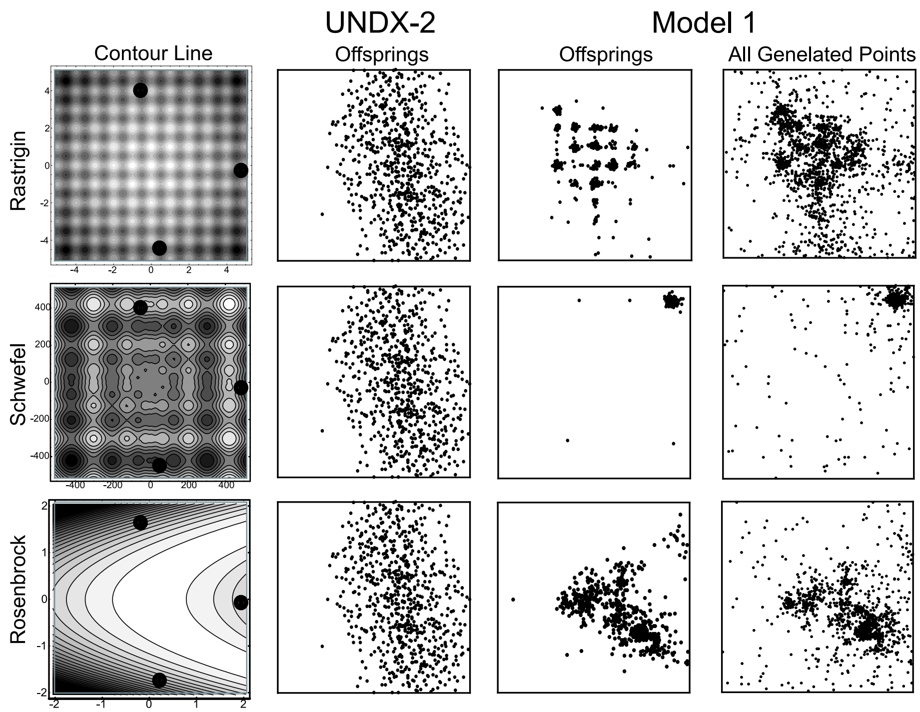

Fig. 5. Distributions of the generated individuals.

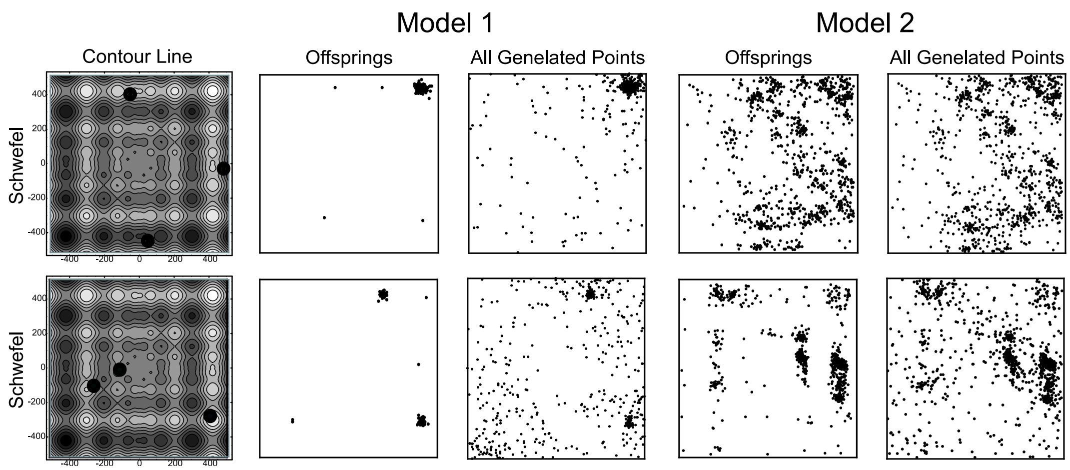

Fig. 6. Distributions of the generated individuals (Schwefel function).

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出典:
2004 IEEE Conference on Cybernetics and Intelligent Systems，CD-ROM Proceedings，2004 年 12 月

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