CONSIDERATION OF SEARCHING ABILITY FOR DISTRIBUTED PROBABILISTIC MODEL-BUILDING GENETIC ALGORITHM

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ABSTRACT
Distributed Probabilistic Model-building Genetic Algorithms (DPMBGAs) are a new type of Genetic Algorithm. In the DPMBGA, when the offspring are generated, Principal Component Analysis (PCA) considers the correlations among the design variables. Moreover, this model applies the island model to maintain population diversity. The effectiveness of DPMBGA has been demonstrated through optimization of continuous functions. This paper describes the effectiveness of the parameters in the DPMBGA. Experiments indicated that these parameters are important factor for maintenance of the diversity of the population.

KEY WORDS
Optimization, Genetic Algorithms and Evolutionary Computation, Distributed Processing Probabilistic Model Building.

1 Introduction

Genetic Algorithms (GA) are stochastic search algorithms based on the mechanics of genetics and natural selection[1]. In GA, the searching point in the search space is considered as an individual in a living population. GA can find an optimal solution after a number of repeats of the operations: selection, crossover, and mutation. In GA, the key issues for effective searching are how to maintain the diversity of the population, treat the correlations among design variables, and to inherit the characteristics of the parent by the child.

Distributed Probabilistic Model-building Genetic Algorithms (DPMBGA)[2] are hybrid methods of Distributed Genetic Algorithms (DGA)[3] and Probabilistic Model-building Genetic Algorithms (PMBGA)[4]. PMBGAs are also called Estimation of Distribution Algorithms (EDA)[5]. The factors of distributed population maintain the diversity of the solutions. In PMBGA, the good characters of parent individuals are inherited by child individuals, because the latter are produced by statistical information of the parent individuals. Moreover, in the DPMBGA, when offspring are generated, Principal Component Analysis (PCA) considers the correlations among the design variables. Therefore, it is thought that DPMBGA has greater search ability. However, there are several parameters in DPMBGA, and the derived solution depends strongly on these parameters. Therefore, the present study was performed to examine the effectiveness of the parameters for search ability.

2 Distributed Genetic Algorithm

The Single Population Genetic Algorithm (SPGA) has two problems: the high calculation cost, and the possibility of premature convergence to local optima. One of the solutions to these problems is to use a Distributed Genetic Algorithm (DGA).

In DGAs, the population is divided into sub-populations (islands). The Genetic Operators are performed in each sub-population. Therefore, DGA is also called the Island model. Moreover, DGAs inlcude the operation called migration in which some individuals are transferred to other islands in every certain generation. The Interval of migration and Rate of individuals by migration are called Migration Interval and Migration Rate, respectively. DGAs have been reported to be able to find better solutions than SPGAs [3]. In the present study, a DGA was applied to maintain the diversity of individuals.

3 Probabilistic Model-Building Genetic Algorithm

The procedures of the Probabilistic Model-Building Genetic Algorithm (PMBGA) are as follows. First, some individuals with higher fitness values are selected from the population. Then, new search points are generated from the probability distribution of the selected individuals. These new points are substituted with individuals in the main population. This operation is repeated until terminal condition is satisfied.

An overview of PMBGA is shown in Fig. 1.

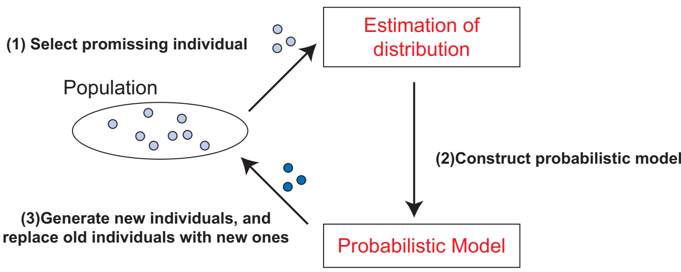

Figure 1. PMBGA

4 Distributed Probabilistic Model-Building Genetic Algorithm

4.1 Flow of DPMBGA

DPMBGAs are a type of DGA and the main population is divided into a number of sub-populations. Operations of PMBGA are performed in each island, and individuals in the islands are transferred to other islands in every certain generation.

An overview of DPMBGA is shown in Fig. 2.

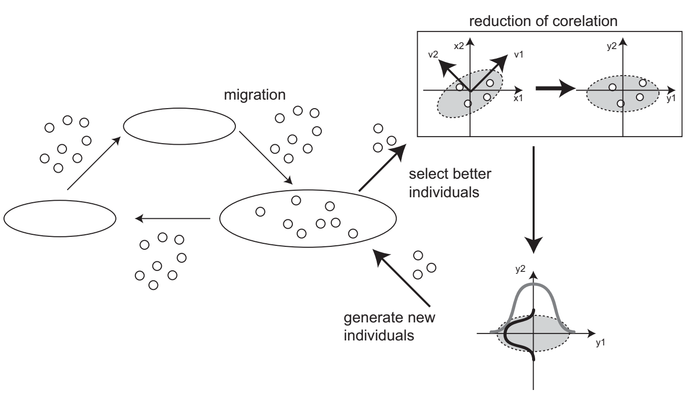

Figure 2. DPMBGA

In the DPMBGA, the following procedures are performed at the generation t.

1. Some individuals are transferred to other island by migration method.
2. The elite individual is reserved.
3. The individuals with good values of fitness are sampled.
4. The above individuals are transferred by the PCA into the new space.
5. New individuals are generated into the new space.
6. New individuals are transferred into the original space.
7. New individuals are substituted for old individuals.
8. The mutation operation is applied.
9. When the reserved elite individuals are eliminated, they are recovered.
10. The new individuals are evaluated.

Each operation is explained in detail in the following sections.

4.2 Migration Methods

In the Migration operation, There are several methods to migrate individuals In the present study, the following two migration methods were used.

- Net Topology
  The most general migration strategy is that of unrestricted migration (complete net topology). Here, individuals may migrate from any island to another. For each island, a pool of potential immigrants is constructed from the other island. The individual migrants are then uniformly at random determined from this pool.
- Ring Topology
  The target island of migration is constructed the ring randomly at each migration chance. The individuals for migration are selected random in each island. The migrated individuals are substituted with individuals that have the worst evaluation values in the island.

4.3 Extraction of sampling individuals

The individuals with higher evaluation values from each island are selected by sampling rate, and are called sampling individuals. Sampling individuals exist in each island. New child individuals are created from the statistics of sampling individuals.

4.4 Reduction of correlations among design variables

The data of sampling individuals are transformed by Principal Component Analysis (PCA). The sampling individuals of PCA are different from sampling individuals. The best individuals generated until the present generation in each island are the targets of PCA individuals, $T(t)$, which is called the archive of the best individuals (Fig. 3).

The covariance matrix, S, which is a real symmetric matrix, is calculated using this archive. S is derived as follows where N_T is number of sampling individuals.

$$S = \frac{1}{N_T - 1} T^T T \tag{1}$$

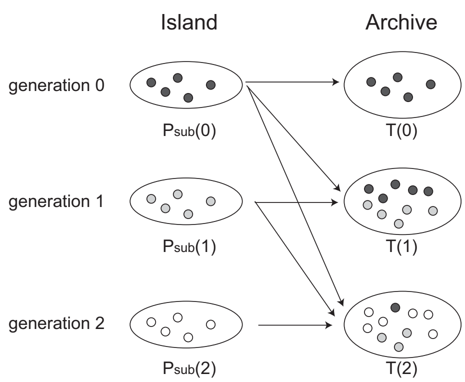

Figure 3. PCA with the archive of the best individuals

The eigenvector is then calculated from the covariance matrix. This eigenvector is used to reduce the correlations among design variables of extracted individuals. An overview of this operation is shown in Fig. 4.

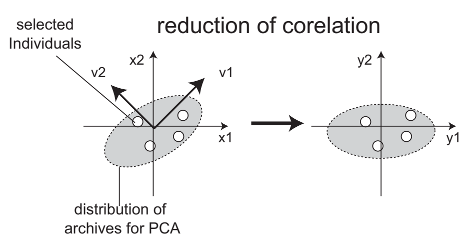

Figure 4. Reduction operation of correlation among design variables with PCA

4.5 Generation of new individuals

New child individuals equivalent to the number of individuals in each island are generated. According to the distribution of individuals with reduced correlation, each individual is created independently by the normal distribution. The normal distribution is multiplied by Amp, and the generated child individuals are expanded.

4.6 Mutation

In the Mutation operation, the values of design variables are changed randomly by the mutation rate. In the present study, the following two mutation methods were used.

- Uniform mutation
  New values of design variables are generated by uniform random numbers within the feasible area.
- Boundary mutation
  New values of design variables are generated on the boundary within the feasible area. In a Real-Coded GA, a mutation method is required that creates individuals on the boundary, because there is a low probability of creating such individuals.

5 Numerical Experiment

The searching ability of the proposed method was examined through optimization of the numerical test functions.

5.1 Test Function

This study used the following four test functions: Rastrigin, Schwefel, Rosenbrock, and Ridge functions in 20 dimensions. There are no correlations among the design variables in the Rastrigin function or the Schwefel function, and these functions have many sub-peaks in their landscapes. On the other hand, there are correlations among the design variables in the Rosenbrock function and the Ridge function, in which there is only a single peak in the landscape.

$$\begin{aligned} F_{Rastrigin}(x) &= 10n + \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i)\right) \\ & \quad (-5.12 \leq x_i < 5.12) \end{aligned} \tag{2}$$

$$\begin{aligned} F_{Schwefel}(x) &= \sum_{i=1}^{n} -x_i \sin\left(\sqrt{|x_i|}\right) \\ & \quad (-512 \leq x_i < 512) \end{aligned} \tag{3}$$

$$\begin{aligned} F_{Rosenbrock}(x) &= \sum_{i=1}^{n-1} \left(100(x_{i+1} - x_i^2)^2 + (1 - x_i)2\right) \\ & \quad (-2.048 \leq x_i < 2.048) \end{aligned} \tag{4}$$

$$\begin{aligned} F_{Ridge}(x) &= \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2 \\ & \quad (-64 \leq x_i < 64) \end{aligned} \tag{5}$$

5.2 Parameters of DPMBGA

The initial parameters of DPMBGA in the numerical experiment are shown in Table 1.

Table 1. Initial parameters of DPMBGA

Population size	512
Number of islands	32
Number of elite	1
Migration interval	5
Migration rate	0.625
Archive size for PCA	100
Sampling rate	0.25
Amp of Variance	1.5
Mutation rate	0.1/(Dim. of functions)
Maximum generation	1000

5.3 Consideration of migration methods

Migration is one of the important functions to maintain diversity. This research considers two migration methods. One of migration methods is ring topology, and other is net topology.

Table 2 shows the number of optimal solutions reached by the two migration methods. We define the optimal solution as an evaluation value of 10^{-10}.

Table 2. Number of optimal solutions reached by Migration methods

	Ring topology	Net topology
Rastrigin	19	0
Schwefel	14	4
Rosenbrock	20	20
Ridge	20	20

As shown in Fig. 2, the ring topology have higher searching ability than the net topology in Rastrigin and Schwefel function. On the other hand, there is no difference in Rosenbrock and Ridge function. The reason is that Rastrigin and Schwefel is the functions which have many sub-peaks in their landscapes. The ring topology can maintain the diversity, because difference space is searched in each island. On the other hand, the net topology transfer individual to all islands, so it cannot maintain the diversity well. Therefore we concluded that the ring topology has higher ability to maintain the diversity.

5.4 Consideration of amplification of distribution

This research considers the amplification of distribution parameter (Amp). In DPMBGA, child individuals are created using the statistic information of the normal distribution of individuals in the present generation. When the parameter of Amp is small, the space for creating new individuals becomes narrow. Conversely, when Amp is large, it becomes wider. Therefore, Amp is one of the most important parameters for DPMBGA. Fig. 5 shows an overview of Amp.

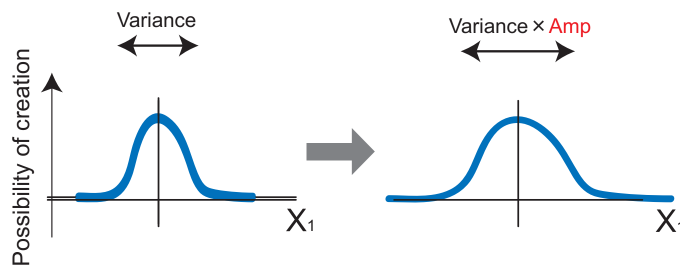

Figure 5. Overview of amplification of distribution

This research considers the number of times that the optimal solution is derived within 20 trials when Amp is changed from 1.0 to 3.0 at intervals of 0.1. The definition of an optimal solution is the same as in the previous experiment. Fig. 6 shows the number of times that the optimal solution was reached along with changes in Amp. The vertical line shows the number of optimal solutions discovered and the horizontal line shows the Amp. The results of the Rastrigin function, Schwefel function, Rosenbrock function, and Ridge function are shown in the upper left, upper right, lower left, and lower right, respectively.

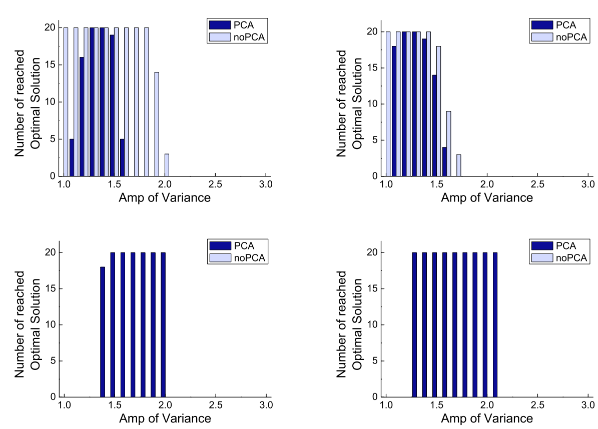

Figure 6. The number of optimal solutions reached by amplification of distribution

Fig. 7 shows the distribution of design variables and transition of evaluation values of the Rastrigin function.

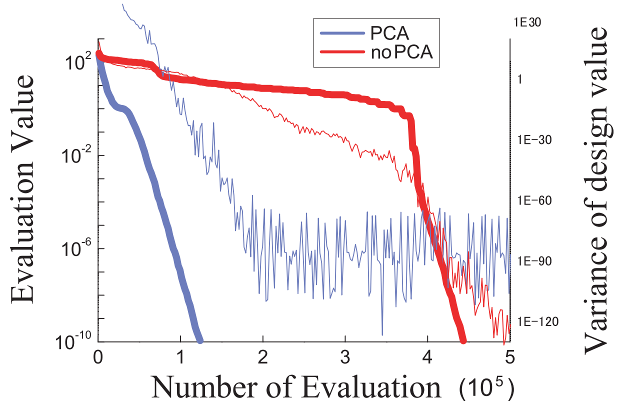

Figure 7. Transition of evaluation value by amplification of distribution

As shown in Fig. 6, only DPMBGA with PCA can find the optimal solution in Rosenbrock and Ridge functions, which have correlations among the design variables. However, Fig. 6 confirms that this model has lower search ability than the model without PCA in Rastrigin and Schwefel functions that do not have correlations among the design variables. The PCA transformation breaks the statistical information of parent individuals in problems that have no correlations among design variables. Fig. 7 illustrates that the transition of evaluation value without PCA reaches the optimal solution rapidly. However, the model using PCA takes a great deal of time to find the optimal solution.

5.5 Consideration of Mutation Method

In DPMBGA, individuals of the next generation are created from statistical information of the parent individuals. However, the mutation method can break the statistical information. Therefore, in this study, mutation methods were verified using the Uniform mutation, the Boundary mutation, and the model without mutation.

Table 3 shows the number of optimal solutions reached by the three mutation methods. The definition of an optimal solution is the same as in the previous experiment.

Fig. 8 shows the transition of the evaluation value and distribution of design variables in the Rastrigin function.

As shown in Table 3, all mutation methods reach the optimal solution in the Rosenbrock function and Ridge function, which have dependence among design variables. However, only Uniform mutation could find the optimal solution in Rastrigin and Schwefel functions that do not have correlations among the design variables. The Uniform mutation may have a mechanism to break the statistical information of the individual.

Table 3. Number of optimal solutions reached by three Mutation methods

	Without	Uniform	Boundary
Rastrigin	0	19	0
Schwefel	0	14	0
Rosenbrock	20	20	20
Ridge	20	20	20

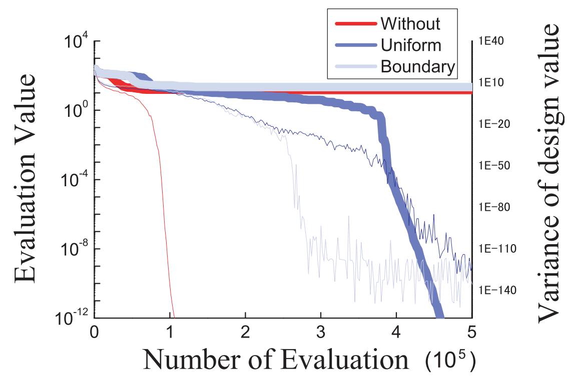

Figure 8. Transition of evaluation value by mutation methods

Moreover, Fig. 8 confirms that Uniform mutation maintains the diversity of the main population. In PCA operation, child individuals are created in the neighborhood of the parent individuals. Therefore, DPMBGA must maintain diversity in problems that do not have correlations among design variables.

5.6 Consideration of sampling rate

In DPMBGA, individuals are selected from the main population according to the sampling rate and the PCA transformation is performed on the individual. The child individuals are generated according to the statistical information of the selected individuals. Therefore, if the number of individuals is small, the population will lose diversity. However, if the number of individuals is too high, it will no be possible to generate a better individual because it will use the statistical information of inferior individuals. Therefore, an appropriate sampling rate is very important to improve the search ability.

Table 4 shows the number of trials that derived the optimal solution along with the sampling rate. The definition of optimum solution is the same as in the previous experiment.

Fig. 9 shows the transition of evaluation value and distribution of evaluation value of the Rastrigin function.

From Table 4, we can see that the best result is obtained when the sampling rate is 0.25 (4 individuals). With

Table 4. Number of optimal solutions reached by sampling rate

	0.125	0.25	0.375	0.5	0.75	1.0
Rastrigin	0	19	0	0	0	0
Schwefel	0	14	0	0	0	0
Rosenbrock	0	20	20	0	0	0
Ridge	0	20	20	20	0	0

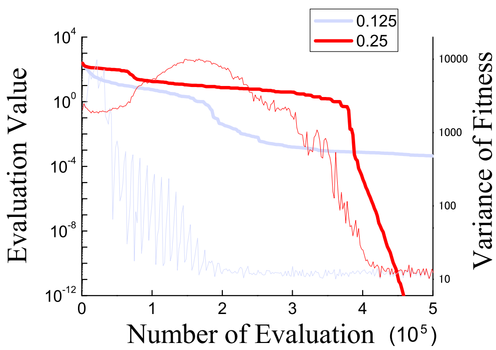

Figure 9. Transition of evaluation value by sampling individuals

a sampling rate of 0.125, statistical information is derived only from two individuals. It is not sufficient to collect the statistical information of parent individuals.

When the sampling rate is high, the statistical information of inferior individuals is used. Therefore, it could not obtain better statistical information and this procedure will be incapable of generating better individuals.

As shown in Fig. 9, when the sampling rate is low, the diversity converges quickly in the case of the Rastrigin function. When the sampling rate is high, convergence cannot be seen. Therefore, for problems that do not have correlations among design variables, it is necessary to maintain the diversity appropriately, because this will strongly affect the search ability.

6 Conclusions

This study examined the searching ability of DPMBGA. The PCA operation of DPMBGA can reduce the correlations among design variables, while a probabilistic model of individuals is constructed. Moreover, the Island model is also adapted to maintain the diversity of the solutions. This paper discussed the parameter effects: i.e., amplification of distribution, mutation method, and sampling individuals. The results of experiments indicated that DPMBGA using PCA showed higher search ability in problems that have correlations among design variables. On the other hand, it is possible to lose the diversity of the population by PCA in problems that do not have correlations among design variables. Therefore, we conclude that parameters to maintain the diversity of the population are important factors to resolve these problems of DPMBGA.

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