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An Improved Stress-based GA for Multi-constrained Topology Optimization

Cuimin Li[1]*, Hiroyasu Tomoyuki[2], Mitsunori Miki[2]

[1] *Graduate School of Engineering, Doshisha University, Kyotanabe, Kyoto, 610-0321, Japan*
[2] *Department of Engineering, Doshisha Unversity, Kyotanabe, Kyoto, 610-0321,Japan*
e-mail: licuimin@mikilab.doshisha.ac.jp, tomo@is.doshisha.ac.jp, mmiki@mail.doshisha.ac.jp

Abstract Genetic algorithms (GAs) for structure topology optimization problems (STOPs) have been developed in recently because GAs are flexible and effective to be applied to various complicated engineering problems. A stress-based crossover (SX) operator [1] for continuous STOPs was proposed to suppress the "checkerboard" pattern and disconnection phenomena, which are common for simple GA for STOPs. Here, this SX operator was improved and the details are described. Different generation models were adopted to verify the effectiveness of this operator. For GA to multi-constrained STOPs, how to define fitness function is always an important consideration because the fitness value determines which individuals maybe transmitted to the next generation. The fitness function used in this paper is well defined to compose the objective function and constraints items. These discussions were examined through a number of multi-constrained STOPs. The results demonstrated that improved SX with GA is a global search algorithm with which it is easy to obtain the applicable topology.

Key words: GA, Stress-based Crossover, Fitness Function, Topology Optimization, Structure Optimization

INTRODUCTION

Structural topology optimization involves searching for an optimal material layout in engineering. Formal methods addressing this problem include the homogenization method [2] in which each element in a grid contains composite material of continuously variable density [0,1] and orientation. However, evaluation of optimal microstructures and their orientations is usually cumbersome and numerically complicated [3]. As an important alternative approach within this family, the power-law approach [3], which is also called the SIMP (Solid Isotropic Microstructure with Penalization) method and was originally introduced by Bendsoe [4], has become generally accepted in recent years because of its computational efficiency and conceptual simplicity [5]. However, it does not directly address the original 0-1 problems [6] and thus tends to converge to a local optimal topology with indistinct boundaries or undesirable checkerboard patterns [6,7] or to converge to an infeasible solution to the original 0-1 problems [8]. Xie and Steven proposed the evolutionary structural optimization (ESO) method, which follows the concept of gradual removal of inefficient material from a structure [9]. As an extension of the ESO method, bi-directional ESO (BESO) allows efficient materials to be added in addition to removal of inefficient materials to remedy the elements deleted in previous processes [10]. However, ESO is not an optimization method. Therefore, the results of ESO do not satisfy the objective nor constraint conditions [11,12]. Evaluation optimization algorithms, such as genetic algorithms (GA), simulated annealing (SA), evolution strategy (ES), and evolution programming (EP) to STOPs have been developed. GA is a powerful global stochastic search method and has been applied to a variety of complicated engineering design problems. For STOPs, GA with Binary genotype [13] and simple operators, such as 1-point/2-point crossover operators, often derive a "disconnected topology" or "checkerboard" pattern [14], which makes the resulting topology impractical. Some researchers have attempted to resolve these problems through different representations. For example, the voronoi-based representation introduced by Schoenauer [15], graph representation introduced by Wang and Tai [16], and morphological representation proposed by Tai and Chee [17]. In this paper, we introduce an improved stress-based crossover operator (SX) with GA to multi-constrained STOPs. Different generation evolution models are adopted to validate the effectiveness of this operator. For GA to multi-constrained STOPs, how to define the fitness function is always an important problem. In this paper, we also introduce our defined fitness function in detail. A number of experiments represent the capability of SX with GA to STOPs.

GA TO STRUCTURE TOPOLOGY OPTIMIZATION

GA to structure topology optimization includes the following steps:
(1). Randomly initial population P(t) generation
(2). Structure analysis of P(t)
(3). Fitness calculation of P(t)
(4). Selection operation
(5). Crossover operation
(6). Mutation operation
(7). Structure analysis of child population P'(t)
(8). Fitness calculation of $P^{'}(t)$
(9). Recombination of P(t) and $P^{'}(t)$ to generate next population P(t+1)
(10). If termination, finish. Else go to (4).

First, the initial population is generated randomly. Then, structure analysis is performed for each individual. The fitness of each individual is calculated. Selection, crossover, and mutation operations are carried out to generate new offspring. After structure analyses, recombination selection is applied on P(t) and $P^{'}(t)$ to generate the next population P(t+1). In the following section, we introduce each of the main operators in the GA in detail.

Chromosome Representation When GA is applied to continual STOPs, the design domain is usually divided by fixed regular meshes to describe the material distributaries. Each mesh represents one gene on the chromosome. The distribution of material and voids in the design domain, in which '1' represents material and '0' represents void, is shown in Fig.1. The chromosome length is equal to the mesh size of the design domain. The straightforward and natural representation method is called bit-string or bit-array. In this paper, the bit-string representation is adopted.

1	1	1	0	0
0	0	1	1	1
0	0	0	0	1
0	0	1	1	1
1	1	1	0	0

Fig. 1 Chromosome Representation

Improved Stress-based Crossover Operator One reason for the disconnected phenomenon is that neighbor mesh continuity is not considered correctly. For real problems, the structure properties of the neighboring material, such as stress and stiffness, often change gradually. The ESO method, which can obtain a connected topology, is also based on this principle. Therefore, in this paper an improved stress-based operator is introduced. The procedures of this operator are as follows. First, the nomenclature used in this operator is explained.

$P(t) = \{p_i(t) \mid i \in \{1 \ldots n\}\}$ is population of generation t, n is population size.

$p_i(t)$ is one chromosome.

$p_i(t).code[k] \in \{0,1\}$ is one bit of chromosome.

$p_i(t).weight$ is number of "1" in chromosome.

$p_i(t).stress[k]$ is stress of gene [k].

$p_i(t+1).power[k]$ is power of gene-k of child individual $p_i(t+1)$.

1. Randomly select two individuals $p_i(t)$,$p_j(t)$ from P (t).
2. Add the stress value at each gene of p_i and p_j by formula (1). Naming this value as the power of each gene of child individual $p_i(t+1)$.

$$p_i(t+1).power[k] = p_i(t).stress[k] + p_j(t).stress[k] \ ,k = 1\ldots N \tag{1}$$

3. Sort the power values of $p_i(t+1).power[k]$ from large to small.
4. Divide the genes of $p_i(t+1)$ into two groups, U0 and U1. U1 is the group of genes of which the power value is in the front-*m* of the sorted genes. U0 is the last group of genes. Here, *m* is defined by formula (2). A child individual $p_i(t+1)$ is generated by formula (3).

$$p_i(t+1).weight = (p_i(t).weight + p_j(t).weight)/2 \tag{2}$$

$$p_i(t+1).code[k] = \begin{cases} 1 & , if\, p_i(t+1).power[k] \in U1 \\ 0 & ,if\, p_i(t+1).power[k] \in U0 \end{cases} \tag{3}$$

Mutation Operator After crossover operation, the mutation operator is applied to each gene of each individual with a small rate. The mutation operator focuses on local search. Randomly decreasing "1" in the chromosome drives a lighter topology.

CONSTRAINT HANDLING STRATEGY AND FITNESS FUNCTION DEFINITION

Optimization is defined to find the minimum of the objective function with some inequality-constrained function and some equality functions. Most practical STOPs have some constraints. In this paper, the objective function is defined to minimize the weight subject to constrained stress and constrained displacement as in formula (4), where $Stress_{\lim}$ and $Disp_{\lim}$ are the constrained stress and constrained displacement, respectively. $Stress_{\max}$ is the maximum stress of the topology. $Disp_{\max}$ is the maximum displacement of the topology. In this paper the maximal displacement is replaced by the displacement of loading point.

$$\begin{aligned} &\min.\, f(x) = \sum_{i=0}^{N} x_i \ , \quad x_i \in \{0,1\} \\ &\text{subject to}: Stress_{\max} < Stress_{\lim}, Disp_{\max} < Disp_{\lim} \end{aligned} \tag{4}$$

Penalty functions are often used for constraint handling techniques. In our study, the fitness function is defined as formula (5). The smaller the function values, the better the individual:

$$fitness(x) = \begin{cases} weight + \dfrac{Stress_{\max}}{Stress_{\lim}} + \dfrac{Disp_{\max}}{Disp_{\lim}} + \dfrac{perimeter}{\alpha \times weight} & ;\textit{ For feasible individuals} \\ weight^* + \dfrac{Stress_{\max}}{Stress_{\lim}} + \dfrac{Disp_{\max}}{Disp_{\lim}} + \dfrac{perimeter}{\alpha \times weight} & ;\textit{ For infeasible individuals} \end{cases} \tag{5}$$

In formula (5), *weight* is the "1" number of topology, *weight** is equal to the mesh number of the design domain. That is, this topology is a full solid material topology. The *perimeter* is the length of the geometric topology outline. α is a constant. This fitness function is composed of four items. The first item represents the objective function, the second is the ratio of maximal stress and constrained stress, the third is the ratio of maximal displacement and constrained displacement, and the fourth denotes the contour length of the geometric topology. In our study, the mesh connection is defined such that there is a shared edge or vertex between two meshes as shown in Fig2. Constant α is assigned a value of 4 in this paper.

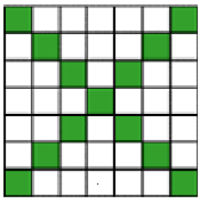


Fig.2 Mesh Connectivity

From the fitness definition, we can see that each of the last three items is less than 1 for feasible individuals. Therefore, the fitness function is focused on the objective function. Once the individuals violate the constraints, the *weight* is assigned the mesh size of the design domain. This guarantees that fitness values of feasible individuals are better than those of infeasible individuals.

GENERATION EVOLUTION MODEL

For GA, the selection operator usually includes survival selection that controls which individual has the opportunity to breed a child, and recombination selection that controls which individuals can be reserved in the next generation. The survival selection method used in this paper is tournament selection. For recombination selection many types of generation evolution models have been proposed.

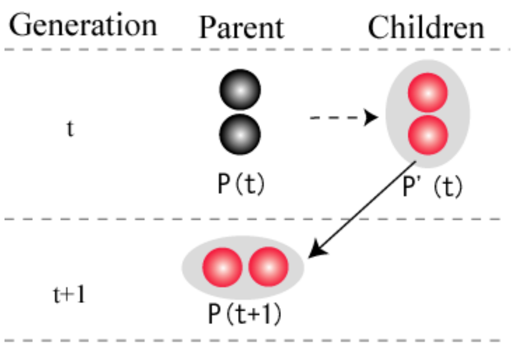


Fig3-1. SGA Model

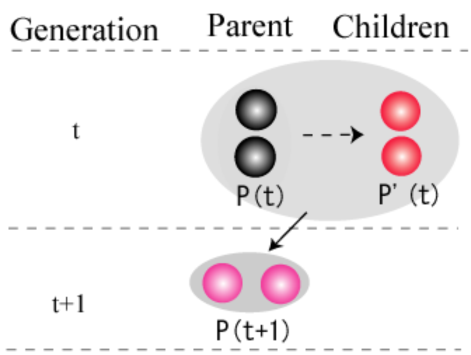


Fig.3-2. ER Model

In this paper, a simple genetic algorithm model (SGA), as shown in Fig.3-1, and elite recombination model (ER), as shown in Fig.3-2, were adopted to validate the effectiveness of SX. For one generation, first randomly select two parents P(t), then use the crossover operator and mutation operators to generate two child individuals P'(t). The SGA model is defined such that the parent population is wholly replaced by the child population. The child population will be the next population. The ER model is defined such that the better two individuals among parent and children will be transmitted to the next generation.

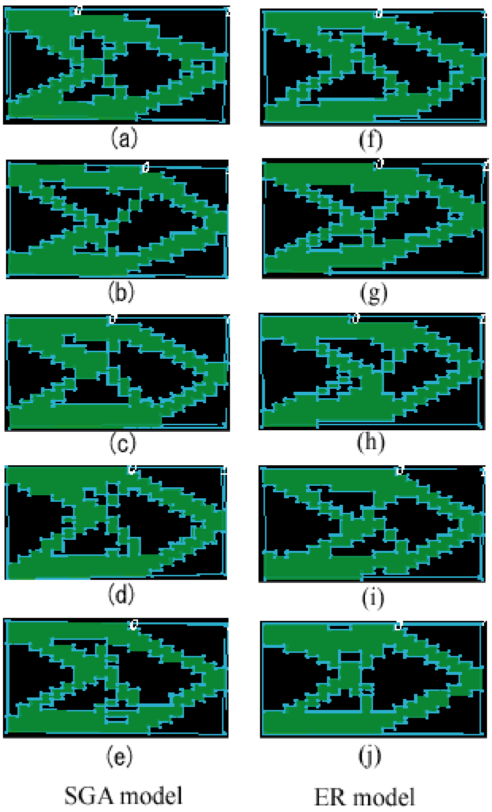
EXPERIMENTS AND DISCUSSION

Generation Evolution Model Discussion - 2D Cantilever Problem A 2D cantilever problem was performed to test the effectiveness of SX with GA by comparison of the SGA model with the ER model. The cantilever dimensions are 20×10 (mm). It is simply fixed at its left, and a downward concentrated load F= 1.0×10^{10} (N) is applied at mid-span on the right frame. For this example, the design domain is divided into 20×20=400 meshes, i.e, the chromosome length is 400. The constrained stress, $Stress_{\text{lim}}$, is 3.5×10^{7} (N) and constrained displacement, $Disp_{\text{lim}}$, is 7 mm. The following material properties are assumed: Young's modulus $E_1 = 206GPa$, Poisson's ratio $\nu = 0.3$, and density $\rho = 1000kgm^{-3}$. The GA parameters are listed in Table 1, and the results of SGA model and ER model for 5 trials are shown in Fig.4.

Among the results (f) is the smallest weight with the maximal stress 3.35004e+10 (N) and maximal displacement 6.81244 (mm). The results show that all the experiments searched out the same resulting topology with the maximal stress and maximal displacement meeting the constraints that demonstrated the capability of SX with GA to STOPs. The geometric results show that there are no marked differences between the SGA model and the ER model. For 5 trials, the average weight of SGA model is 190; the average weight of ER model is 185. Average numerical weight of solution of ER model is better than that of the SGA model.

Table 1. Parameters of GA

Population Size	Chromosome Length	Elites	Crossover Rate	Mutation Rate	Tournament Size	Max. Generation
100	400	1	1	0.01	2	500



index	weight	$Stress_{max}$	$Disp_{max}$
(a)	191	3.33762e+10	6.57782
(b)	189	3.19898e+10	6.77325
(c)	194	3.07069e+10	6.50167
(d)	191	3.36011e+10	6.47777
(e)	186	3.1791e+10	6.72296
(f)	183	3.35004e+10	6.81244
(g)	185	3.36892e+10	6.81551
(h)	186	3.49204e+10	6.75836
(i)	185	3.42326e+10	6.66957
(j)	187	3.28366e+10	6.81041

Fig.4 Results of SGA model and ER-model

Evolution Procedure Discussion - MBB Beam Problem MBB beam with dimensions of 2000×400 (mm) as shown in Fig.5 are used to test the fitness function and evolution procedure of SX with GA. This problem is related to the design of the floor plane of a passenger airplane. The design domain is supported at its ends as shown in Fig.5. The domain has a downward concentrated load $F = 5.12\times10^9$ (N) at mid-span on the upper frame. The frame is divided by 40×16 meshes, i.e, the chromosome length is 640. The objective function is to minimize the weight of the structure with constrained $Stress_{\lim} = 3.3\times10^9$ (N)and constrained displacement $Disp_{\lim} = 0.33$ (mm) respectively. The material properties are the same as in the 2D cantilever problem. GA parameters are the same as in the 2D cantilever problem besides chromosome length.

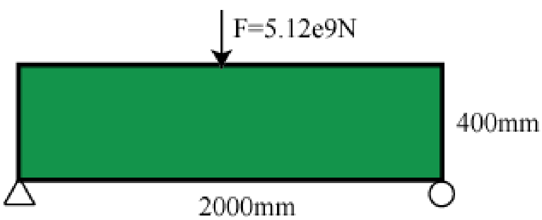
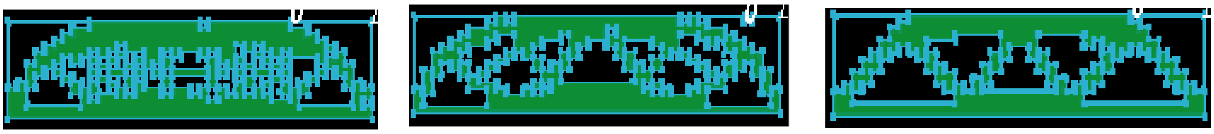


Fig.5 MBB beam problem

Fig. 6 shows the best results of generation 16, 52, and 500. The weights are 60%, 50%, and 47% of the full material structure, respectively. In 16th generation, SX with GA can quickly remove materials of less stressed meshes. This search leads to the optimal outline of the final design. In the 52nd generation, it has found an applicable topology with the numerical results meeting the constraints. As GA uses multiple individuals to search the design domain, it can perform a further optimization until the population loses diversity. For this problem, it searches out a more optimal solution. Comparison of Fig. 6 (b) and (c) indicates that Fig. 6(c) is smaller on weight has less difference between maximal stress and minimal stress, and similar maximal displacement value.



(a) generation=16 (b) generation=52 (c) generation=500

index	weight	$Stress_{max}$	$Stress_{min}$	$Disp_{max}$
(a)	386(60%)	2.87732e+07	6.76895e+05	0.293523
(b)	318(50%)	3.03170e+07	4.06717e+04	0.329299
(c)	302(47%)	2.56965e+07	1.05075e+06	0.325971

Fig. 6 Results of MBB Beam problem

Different Constraints Experiments - Michell Type Problem The Michell type structure is the first truss solution of least weight and is based on a general theory. It has been widely used as a typical problem to verify the effectiveness of evolutionary approaches to STOPs. Hence, in this paper, it is also adopted to test the capability of SX with GA to multi-constrained STOPs. The design domain of dimensions 10000×5000 (mm) shown in Fig.7 is divided into 20×40=800 meshes. The two corners at the bottom are fixed and a downward concentrated load F = 1000N is applied at mid-span on the under frame.

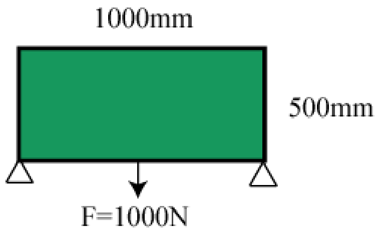
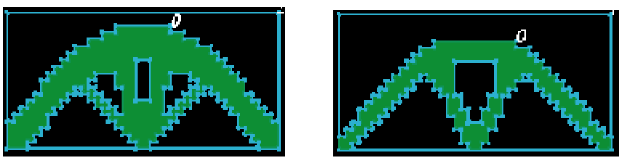


Fig.7 Michell type problem

For this problem, we performed experiments with two group constraints : (a) $Stress_{lim}$ =0.05 (N), $Disp_{lim}$ =1.0×10^{-9} (mm) and (b) $Stress_{lim}$ =0.055 (N), $Disp_{lim}$ =1.5×10^{-9} (mm). The results of experiments (a) and (b) are shown in Fig. 8.



(a) (b)

index	weight	$Stress_{max}$	$Stress_{min}$	$Disp_{max}$
(a)	316	0.0499904	0.0067776	9.95653e-10
(b)	204	0.0549431	0.0106312	1.46536e-09

Fig.8 Results of Michell Type Problems

The geometric results shown in Fig. 8 (a) indicate that GA with SX searches out a topology the same as the theoretical solution. The numerical results meet constraints. When we set a different constraint (b), an applicable solution was also searched out with numerical values close to the constraints.

CONCLUSIONS

GA has been developed to solve STOPs because of its global stochastic search ability and its flexibility for various optimization problems. Crossover is the most important operator that controls global evolution direction. However, 1-point/2-point crossover operator with GA to continuous STOPs often derives to disconnected phenomenon or "checkerboard" pattern. To obtain an applicable topology, an additional strategy must be adopted to suppress the checkerboard pattern or eliminate the disconnected phenomenon. ESO is one formal approach to STOPs with which it is easy to obtain a connected geometric topology. However, ESO is not a search method in the strict sense because it simply removes less stressed materials gradually. Furthmore, it is not fitted for multi-constrained problems. In this paper, we introduced an improved stress-based crossover operator. To verify the effectiveness of this operator, we made a comparison of two different evolutionary models, the SGA model and ER model. Both numerical results and geometric results demonstrated the capability of GA with SX for multi-constrained STOPs. The fitness function definition is important for GA to multi-constrained STOPS, because it determines which individuals may be transmitted to the next generation. In this paper, we defined a fitness function that is composed of objective function and constraints. For feasible individuals, the fitness function focuses on the objective function and the impact of constraint items is slight. Experiments demonstrated the applicability of the fitness function. In addition, the evolution history of the MBB beam problem demonstrates the global search ability of GA with SX. A typical Michell type problem was also adopted to experiment SX with GA on different constraints. The experimental results demonstrated that SX with our defined fitness function is a flexible method for multi-constrained STOPs and is powerful for search out applicable geometric topology.

REFERENCES

[1] Cuimin Li, Tomoyuki Hiroyasu, Mitsunori Miki. *A Hybrid Algorithm for Structural Topology Optimization*. International Conference on Computational Methods 2007.Hiroshima, Japan, 2007.

[2] Bendsoe, M. P., & Kikuchi, N. (1988). *Generating optimal topologies in structural design using a homogenization method*. Computer Methods in Applied Mechanics and Engineering, 197-224.

[3] O. Sigmund, *A 99 line topology optimization code written in MATLAB*, Structural and Multidisciplinary Optimization 21(2), pp. 120-127, 2001.

[4] M. P. Bendsoe, *Optimal shape design as a material distribution problem*, Structural Optimization 1, pp. 193-202, 1989

[5] G. I. N. Rozvany, M. Zhou, and T. Birker, *Generalized shape optimization without homogenization*, Structural Optimization 4, pp. 250-254, 1992.

[6] M. P. Bendsoe and O. Sigmund, *Topology Optimization: Theory, Methods and Applications*, Springer-Verlag, Berlin, 2003.

[7] G. I. N. Rozvany, *Aims, scope, methods, history and united terminology of computer-aided topology optimization in structural mechanics*, Structural and Multidisciplinary Optimization 21(2), pp. 90-108, 2001.

[8] M. Stolpe and K. Svanberg, *On the trajectories of penalization methods for topology optimization*, Structural and Multidisciplinary Optimization 21(2), pp. 128-139, 2001.

[9] Xie, Y. M., & Steven, G. P. (1992*). Shape and Layout Optimization via an Evolutionary Procedure*. In Proceedings of the International Conference on Computational Engineering Science, Hong Kong University of Science and Technology, Hong Kong.

[10] O.M. Querin,G.P. Steven and Y.M. Xie (1998), *Evolutionary Structural Optimization (ESO) Using a Bi-directional Algorithm*, Engineering Computations, 15(8), 1031-1048.

[11] G. I. N. Rozvany, *Stress ratio and compliance based methods in topology optimization - A critical review,* Structural and Multidisciplinary Optimization 21,pp. 109-119, 2001.

[12] M. Zhou and G. I. N. Rozvany, *On the validity of ESO type methods in topology optimization*, Structural and Multidisciplinary Optimization 21(1), pp. 80-83, 2001.

[13] E. Sandgren, E. Jensen, and J.Welton, *Topological design of structural components using genetic optimization methods, in Sensitivity Analysis and Optimization with Numerical Methods, S. Saigal and S. Mukherjee, eds., Proceedings of the Winter Annual Meeting of the American Society of Mechanical Engineers 115, pp. 31-43,(Dallas, TX, USA), 1990.*

[14] Sigmund O, Petersson J. *Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards*, mesh-independencies and local minima. Structural Optimization 1998; 16:68 – 75.

[15] Schoenauer M. *Shape representation for evolutionary optimization and identification in structural mechanics*. Genetic algorithms in engineering and computer science.pp.443-464.1995.

[16] S. Y. Wang and K. Tai, *Structural topology designs optimization using genetic algorithms with a bit-array representation*, Computer Methods in Applied Mechanic and Engineering, 2004.

[17] K. Tai and T. H. Chee, *Design of structures and compliant mechanisms by evolutionary optimization of morphological representations of topology*, ASME Journal of Mechanical Design 122(4), pp. 560-566, 2000.