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# Parameters Discussion of SX for Structural Topology Optimization

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Stress-based crossover (SX) is a genetic operator for structural topology optimization using the information of stress. This paper discusses three types of SX parameter. First, generation alternation models are used to improve the search ability of genetic algorithms. Second, several different meshes are used to study the mesh dependency of SX. A comparison of evolutionary structural optimization (ESO) and SX is performed on the MBB beam problem. Third, element stress ranking method is adopted to study the impact of element stress on final topology. In addition, different domain division strategy for GA and FEM is introduced to further discuss the element stress influence in SX.

***Key Words***: Stress-based Crossover, Genetic Algorithm, Mesh Size, Structural Topology Optimization, Structure Optimization

## 1. Introduction

Major approaches to continuum structure topology optimization include homogenization (1), solid isotropic microstructure with penalization (SIMP) (2), level-set method (3), evolutionary structural optimization (ESO) (4), and bidirectional evolutionary structural optimization (BESO) (5). Evolutionary computation methods, such as genetic algorithms (GA), multiobjective genetic algorithms (MOGA), and cellular methods, as flexible methods to address various complicated problems, have been extended to solve structure optimization problems.

With regard to application of GA to structural topology optimization problems, checkerboard like material distribution or disconnected structures in resulting topology are the most common problems for structural topology optimization. To solve the disconnected phenomena on geometric solutions, graph representation (6) and morphological representation (7) have been proposed. A number of techniques have been adopted to prevent checkerboard like material distributions, such as smoothing (8, 9), higher-order finite elements (10, 11), and filtering (12). However, smoothing is based on image processing, which ignores the underlying problem (8). Experiments indicated that only higher-order finite element methods with simple GA operators can eliminate the checkerboard like material distribution in the solution (13). However, it is obvious that using higher-order finite element methods will substantially increase computation cost. Filtering methods, which are variations of image-processing techniques, involve modification of the design sensitivities used in each generation of the algorithm. For filtering methods, the design sensitivities of specific elements depend on a weighted average over the element itself and its eight direct neighbors and are very efficient in removing checkerboards (8). However, when filtering methods are applied to three-dimensional problems, realization will be very complicated.

In a previous study, we introduced a stress-based crossover (SX) operator (14) in which the connections of neighboring elements are considered during the procedure. Experiments demonstrated that this operator can easily obtain a solution without the checkerboard phenomena. However, there are a number of parameters and choices of genetic operators the effects of which on the solutions have not been discussed. Therefore, this paper discusses three types of SX parameter.

First, the generation model of GA is discussed. Application of GA to real problems always has the drawbacks of a large seach space, complicated solution landscape,

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and long computation time. Generation alternation models are used to speed up convergence. Second, sensitivity of element size is described. Mesh dependency, which refers to that it cannot obtain qualitatively the same solution for different mesh-sizes or discretizations, is one numerical instability occuring in applications of many topology optimization methods. It is also discussed the mesh dependency of SX for structure topology optimization.

Thirdly, SX uses element stress to decide which elements will be material and which will be void. Therefore, the impact of element stress on final resulting topology is discussed using element stress ranking method. For GA to structural topology optimization problems, GA search space and FEM space are often divided with same numbered meshes. On one hand, a good design needs sufficient accuracy of the FEM for precise evaluation of stress and displacement, which means large number of elements are needed for structure analysis results accuracy. On the other hand, the GA search space increases greatly with large number of meshes. For GA, large search space needs large population and long evolution iteration times to do a global exploration that is not affordable for designers. Therefore, different domain division strategy is introduced for GA and FEM space to further study the influence of element stress in SX.

This paper is organized as follows. Section 2 introduces application of SX to structural topology optimization. Section 3 defines a penalty fitness function. Section 4 presents the generation alternation models. Section 5 discusses the mesh dependency of SX. In Section 6, element stress ranking method and different domain division strategy for GA and FEM are used to study the impact of element stress to final resulting topology. Finally, our conclusions are presented in Section 7.

## 2. SX to Structural Topology Optimization

For application of GA to structural topology optimization, the GA search space is divided by fixed regular meshes. The whole search space is taken as one chromosome and each mesh represents one gene on the chromosome. GA uses "1" and "0" on each gene to describe the material distributions, where "1" represents material and "0" represents void.

On structure analysis side, the material is divided by linear hexahedron that is called "element" in finite element method (FEM). The objective of this method is to obtain rough design configuration and accurate stress evaluation is not required. Therefore, void elements are not really "no material" but assigned a small Young's modulus to evaluate the stress. In this paper, the Young's modulus for void elements is one thousand of Young's modulus for solid elements. The VonMises equivalent stress of each element is alaysed by ADVENTURE[15]. The materials distribution of elements is used to describe the approximate topology.

The initial population is generated randomly. After selection, SX, and mutation operations, offspring individuals are generated. In this section, the procedures of SX are listed in detail. First, the nomenclature used in this operator is explained.

- $P(t)$=$\{p_i(t)|i \in \{1 \dots n\}\}$ is population of generation $t$, $n$ is the population size.
- $p_i(t).weight$ is number of "1" in chromosome.
- $p_i(t).code[k]$∈$\{0, 1\}$ is genotype of individual, where $k$∈$\{1 \dots N\}$, $N$ is chromosome length.
- $p_i(t).stress[k]$ is stress of element $k$.
- ${p'}_i(t).ability[k]$ is ability of gene $k$ of child individual ${p'}_i(t)$.

1. Two individuals are selected randomly without considering the fitness value, $p_i(t)$, $p_j(t)$ from population $P(t)$.
2. Sum the stress at each gene of $p_i(t)$ and $p_j(t)$ by formula(1). This value is designated as the *ability* of each gene of child individual $p'_i(t)$.

$$\begin{aligned} p'_i(t).ability[k] = \quad & p_i(t).stress[k] + p_j(t).stress[k], \\ & k = 1 \dots N \end{aligned} \tag{1}$$

3. Sort the ability values of $p'_i(t).ability[k]$ from large to small.
4. According to $p'_i(t).ability[k]$ , divide the genes into two groups, U1 and U0. The front $m$ genes belong to U1, and the remaining genes belong to U0. Here, $m$ is defined by formula (2). A child individual, $p'_i(t)$, is generated by formula (3).

$$m = [p_i(t).weight + p_j(t).weight]/2 \tag{2}$$

$$p'_i(t).code[k] = \begin{cases} 1, & if\ p'_i(t).ability[k] \in U1 \\ 0, & if\ p'_i(t).ability[k] \in U0 \end{cases} \tag{3}$$

Step 4 is the most important of the four steps. There are two key points for generating new individuals. The first is which elements will be set to "1," and the second is how many elements will be set to "1." This operator defines that elements with large *ability* value will be set to "1." With regard to the second question, because the initial GA population is generated randomly, it will inevitably include some infeasible individuals; indeed, it is even possible that the whole population will be comprised of infeasible individuals. Therefore, at the beginning of evolution, the SX focuses on searching for a feasible individual. Hence, we define "1" elements number of offspring individuals is equal to the average of that of the parent individuals. These four steps are applied on population $P(t)$ to generate offspring individuals.

## 3. Problem Description and Fitness Function

The objective function of all the examples in this paper is to minimize weight subject to constrained stress and constrained displacement, which can be expressed in formula (4).

$$\begin{aligned} min.weight &= \sum_{k=1}^{N} code[k], code[k] \in \{0,1\} \\ subject\_to &: Stress_{max} \leq Stress_{allowable} \\ Displacement &\leq Displacement_{allowable} \end{aligned} \tag{4}$$

For feasible individuals, the fitness function is defined as formula (5). In this function, each of the last three terms is less than or equal to "1." The first term, which is the objective function, is much larger than other terms such that the fitness function focuses on *weight*. The last three terms are penalty functions.

$$\begin{aligned} fitness = \quad & weight + \frac{Stress_{max}}{Stress_{allowable}} \\ & + \frac{Displacement}{Displacement_{allowable}} + \epsilon \end{aligned} \tag{5}$$

In formula (6), $\epsilon$ is defined to indicate the geometric topology influence.

$$\epsilon = \frac{perimeter}{4 \times weight} \tag{6}$$

where *perimeter* is the length of the geometric topology profile as shown in Fig.1. To reduce the influence of this term, it is divided by $4 \times weight$.

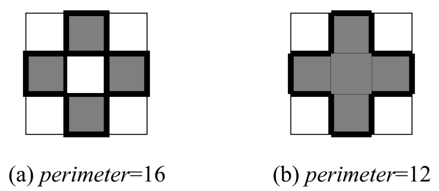


Fig.1 Mesh Connection and *perimeter* Definition

For infeasible individuals, the first item of formula (5) is replaced by a constant, $C$, which must be larger than *weight* of any feasible individual. In this paper, $C$ is equal to meshes number of GA search space.

## 4. Generation Alternation Models

In a simple GA model (SGA), there is no competition between the parents and the children, so the children

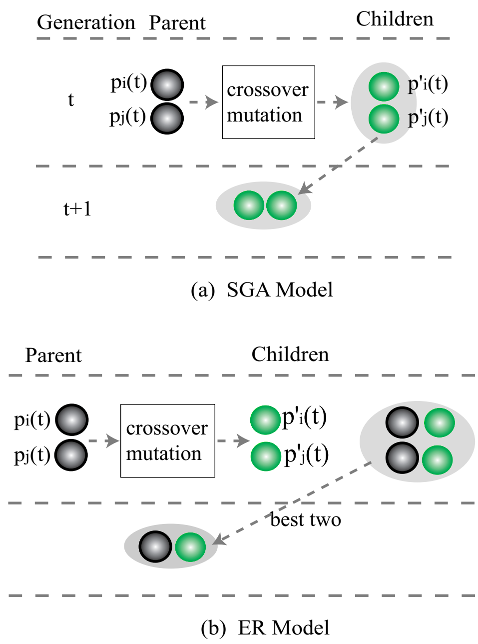


Fig.2 Generation Alternation Model

replace the parents irrespective of their fitness. This selection scheme steers the tendency of the algorithm according to the information obtained in previous steps. Therefore, various generation alternation models have been proposed to improve the search ability of GA, including minimal generation gap (MGG) [16] and elitist recombination (ER) [17]. In this study, the ER model is adopted for comparison with the SGA model to speed up convergence and improve the search ability of GA. The SGA model and ER model are shown schematically in Fig.2.

The SGA model is defined such that the child population replaces the parent population. The child population will be transmitted to the next generation. In contrast, the ER model is defined such that the better of two individuals among parents and children will be transmitted to the next generation.

**4.1 Experiment and Discussion** The 2D cantilever problem, as shown in Fig.3, is a benchmark problem of structural topology optimization that has been used extensively in experiments. Here, this problem is also adopted to allow comparison of the SGA model with the ER model. The dimensions are 20 $(mm) \times 10$ $(mm)$. The thickness is 1 $(mm)$. The beam is simply fixed at its left and a downward concentrated load F $= 1.0$ *times* $10^{10}$ $(N)$ is aplied at the mid-span on the right frame. The design domain is divided into $20 \times 20$ meshes. The constraints are $Stress_{allowable} = 3.5 \times 10^{7}$ $(Pa)$ and $Displacement_{allowable} = 7.0$ $(mm)$.

In this paper, the following material properties are assumed: Young's modulus $E = 206$ $GPa$, Poisson's ratio

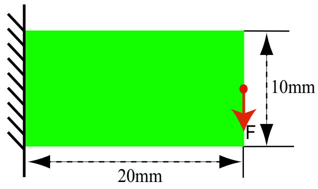


Fig.3 2D cantilever problem

$\nu = 0.3$, and density $\rho = 1000\ kg/m^3$.

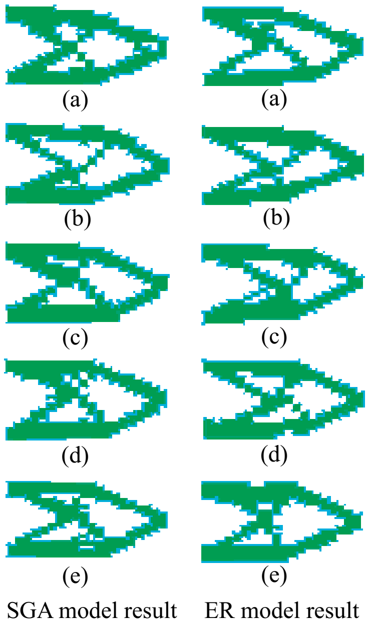


Fig.4 Results of SGA and ER model

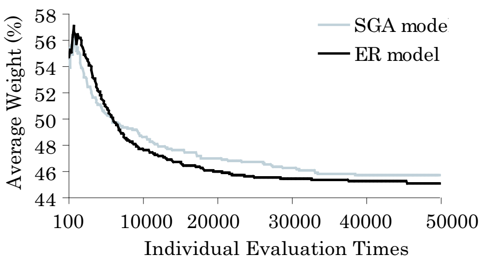


Fig.5 Comparison of *weight* evolution history

For each model, it is run with five trials using different random numbers. The final resulting topologies are shown in Figs. 4. These figures show that there are no marked differences between the results obtained with the SGA model and those with the ER model. For five trials, the average weights of best solution by the SGA and ER models are 190 (47.5%) and 185 (46.2%), respectively. The evolution histories of average weight of best solution for five trials are compared in Fig.5. The beginning 5000 individual evaluation times of evolution histories is detailed in Fig.6. At the beginning of evolution, the ER

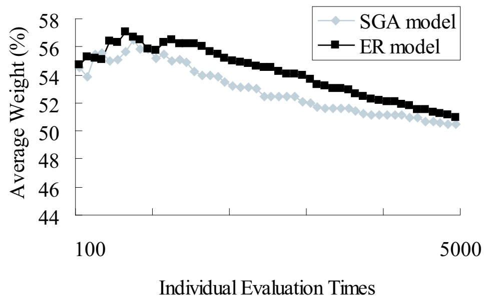


Fig.6 Part of *weight* evolution histories of Fig.5

model is found to search the design domain more widely than SGA model. The convergence speed by the ER model is faster than that by the SGA model.

## 5. Mesh Dependency Discussion

In this section, several different meshes are examined experimentally to discuss the mesh dependency of SX.

For application of GA to real-world engineering problems, different researchers define different fitness functions to evaluate the individuals, which makes it difficult to decide which solution is the optimum. Especially for application of evolutionary computation algorithms to multi-constrained problems, many fitness evaluation approaches have been proposed [18]. According to the above analyses, for application of GA to multi-constrained problems, different geometric topologies may be searched. Hence, for each mesh size, the experiment is run with six trials using the same parameters.

**5.1 Example: MBB Beam Problem** The MBB beam with dimensions of $2000 \times 400$ $(mm)$ is shown in Fig.7. The thickness is $10mm$. The design domain is a simple beam supported at its ends, with a downward concentrated load F=$5.12 \times 10^9 (N)$ applied at the mid-span on the upper frame.

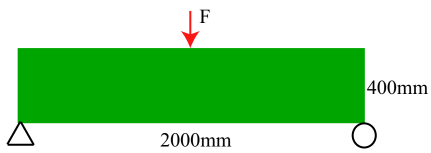


Fig.7 MBB Beam Problem

For this example, the constraints are $Stress_{allowable} = 3.3 \times 10^7 (Pa)$ and $Displacement_{allowable} = 0.33$ $(mm)$.

**5.2 Mesh Size** In this paper, four meshes are used: $(100, 100)$, $(100, 50)$, $(50, 50)$ and $(50, 25)$ $(mm)$. With these meshes, the mesh numbers are $20 \times 4$, $20 \times 8$, $40 \times 8$, and $40 \times 16$, respectively.

**5.3 Experiments on Different Mesh Size**

- Mesh Size (100, 100) $(mm)$

Geometric solutions of six trials for mesh size (100, 100) $(mm)$ are shown in Fig.8.

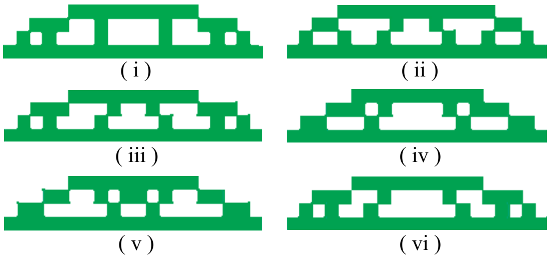


Fig.8 Results of Mesh= $20 \times 4$

- Mesh size (100, 50) $(mm)$

  Geometric solutions of six trials for mesh size (100, 50) $(mm)$ are shown in Fig.9.

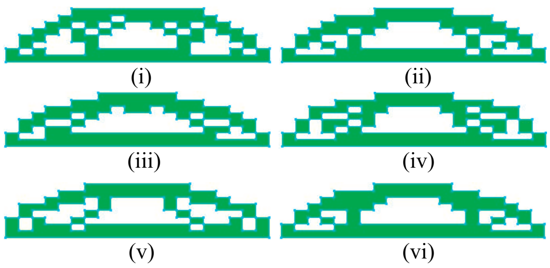


Fig.9 Results of mesh = $20 \times 8$

- Mesh size (50, 50) $(mm)$

  For a mesh size of (50, 50) $(mm)$, the resulting topologies are shown in Fig.10.

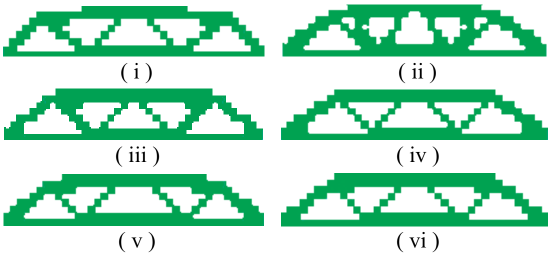


Fig.10 Results of Mesh = $40 \times 8$

- Mesh size (50, 25) $(mm)$

  The resulting topologies of six trials with a mesh size of (50, 25) $(mm)$ are shown in Fig.11.

  .

**5.4 Results Discussion** Topology similarity evaluation is a difficult problem, which refers "hole" number, position and shape in a structure. In this paper, we only study the "hole" number to evaluate the topology similarity. According to this definition, if two structures have the same "hole" numbers they are taken as having the same topology. Comparisons of Fig.8, Fig.9,

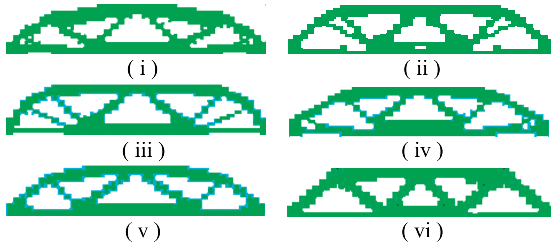


Fig.11 Results of Mesh = $40 \times 16$

Fig.10 and Fig.11 show most of the topologies are similar. Comparison of Fig.8 and Fig.10 demonstrates by square meshes SX can search out a simple topology without well domain division.

**5.5 Comparison of SX with ESO** ESO, as an effective approach to structural topology optimization problems, has been widely used to solve various engineering problems. As this method is also based on stress, a comparison of ESO and SX is performed to further examine the mesh dependency of SX.

One solution of SX for different mesh sizes and ESO results are shown in Fig.12. Accordingly, the numerical properties of each solution are compared in Table 1.

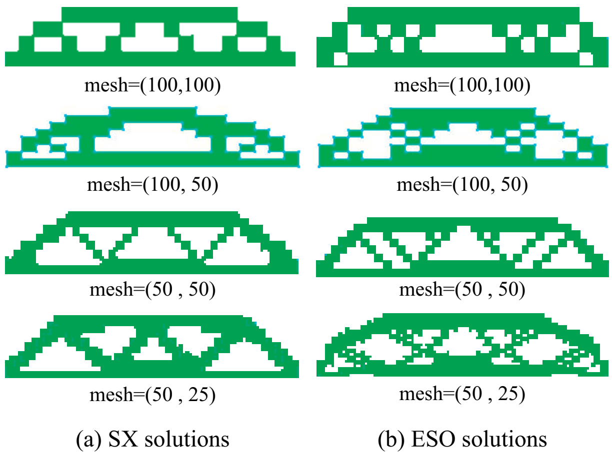


Fig.12 Solution Comparison of SX and ESO

The geometric results comparison of SX and ESO in Fig.12 demonstrates that SX searched out the same topology with different meshes. The numerical results shown in Table 1 indicate that with decreasing mesh size, the structure weight became progressively smaller. In contrast, for ESO, different mesh size yielded different resulting topologies. Furthermore, the structure weight does not decrease with decreasing mesh size. SX results demonstrated that a certain mesh size is sufficient to obtain a feasible solution, and subsequent reductions in mesh size do nothing to derive a more optimal solution. For this MBB beam problem, geometric topology with mesh size (100, 100) $(mm)$ is approximate with ge-

Table 1 Numerical Properties Comparison of Fig.12

| Method | Mesh $(mm)$ | $Weight(\%)$ | $Stress_{max}$ $(Pa)$ | $Displacement$ $(mm)$ |
|---|---|---|---|---|
| SX | (100,100) | 55.0 | 1.576e+07 | 0.310 |
| | (100,50) | 51.8 | 2.589e+07 | 0.325 |
| | (50,50) | 47.5 | 2.621e+07 | 0.326 |
| | (50,25) | 47.2 | 2.569e+07 | 0.325 |
| ESO | (100,100) | 48.1 | 3.145e+07 | 0.327 |
| | (100,50) | 55.0 | 2.347e+07 | 0.307 |
| | (50,50) | 62.5 | 1.587e+07 | 0.271 |
| | (50,25) | 50.3 | 3.292e+07 | 0.328 |

ometric topology with mesh size (50, 50) $(mm)$.

## 6. Discussion of Element Stress in SX

During SX procedures, element stress is used to produce child individuals. In this section element stress ranking method is adopted to discuss the impact of element stress on the resulting topology.

It is well-known that the structure analysis results accuracy is related to finite element size, especially for stress. A good design needs sufficient accuracy of the FEM for precise evaluation of stress and displacement. On the other hand, if the GA search space is divided with detailed meshes the chromosome becomes much long that means large design variables. Usually large design variables need large population and many evolution iteration times to do a global exploration. Actually, because designers cannot afford the long analysis time small population size and short iteration times are often prefered. In this section, the element stress impact of SX is further discussed by different domain division strategy for GA and FEM for the purposes of both reducing GA design variables and increasing the accuracy of FEM.

**6.1 Element Stress Ranking Method** After finite element analysis, the elements are ranked according to element stress value. The element with big stress value is assigned a big rank number as shown in Fig.13. During SX procedures, the element stress vaue is replaced by this rank number. The modified SX procedures 1 and 2 are listed as follows.

1. Randomly select two individuals, $p_i(t)$, $p_j(t)$ from population $P(t)$. Applying element stress ranking method on $p_i(t)$, $p_j(t)$. The rank number of each element is named $p_i(t).rank[k]$.
2. Sum the $p_i(t).rank[k]$ at each gene of $p_i(t)$ and $p_j(t)$ by formula(7) to calculate $p_i'(t).ability[k]$.

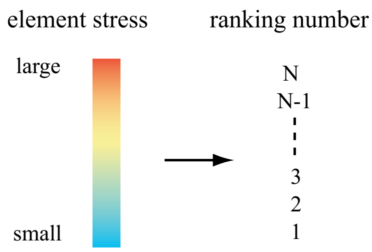


Fig.13 Element Stress Ranking Method

$$p_i'(t).ability[k] = \begin{aligned} & p_i(t).rank[k] \\ & +p_j(t).rank[k], \\ & k = 1 \dots N \end{aligned} \qquad (7)$$

6.1.1 **Experiment** The example is the MBB beam problem with height:length=40 mm:240 mm. The load is 1000$N$. Due to symmetry, only half of the structure is modeled. The half structure is discretized with $40 \times 120$ hexahedrons. The object is to minimize the weight subjected to maximal stress $Stress_{max} \leq 2500$ Pa and maximal displacement $Displacement_{max} \leq 2.0 \times 10^{-6} mm$.

The results by initial SX and element stress ranking method are shown in Figs.14, 15. The numerical properties are listed in Table 2.

Fig.14 SX Solution

The resulting topology shown in Fig.15 has serious checkerboard-like phenomena. However, the general outline profile is similar to Fig.14. About the checkerboard-like phenomena, we suppose it is caused by the lost of

Table 2 Numerical Properties Comparison

| Index | Weight(%) | $Stress_{max}$ $(Pa)$ | $Displacement$ $(mm)$ | Average Stress $(Pa)$ | Variance Stress |
|---|---|---|---|---|---|
| Fig.14 | 48.2 | 2452.10 | 1.97e-06 | 554.23 | 125.10 |
| Fig.15 | 46.9 | 2499.23 | 1.99e-06 | 550.31 | 155.57 |

Fig.15 Solution of Element Stress Ranking

detail difference of elements.

Therefore, we can give these conclusions: the element rank number can determine the general outline profile and the checkerboard-like phenomena are related to the difference of elements.

**6.2 Different Domain Division Strategy for GA and FEM** In this section, element stress influence to the final solution is further discussed through different GA and FEM domain divisions. For example, each GA mesh corresponds to four finite elements in Fig.16. This different domain division strategy for GA and FEM produces two mapping processes, GA→FEM and FEM→GA, between GA domain and FEM domain as the big arrows in Fig.17.

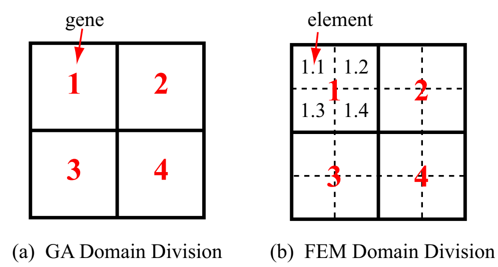


Fig.16 Example of Different Domain Division strategy for GA and FEM

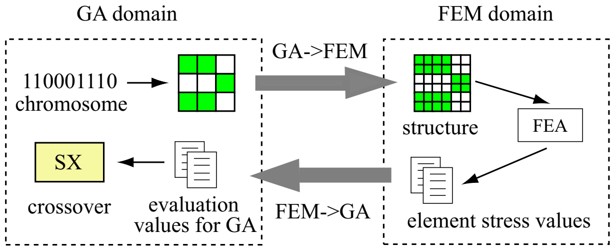


Fig.17 Mappping Processes between GA and FEM

6.2.1 **GA→FEM** Each gene of GA domain is mapped to several elements of FEM domain to prepare for finite element analysis(FEA) for each structure. For this mapping technique, it is defined that : If one gene of chromosome is "1" the corresponding FEM elements are all set materials. On the contrary, if one gene of chromosome is "0" the corresponding FEM elements are all set void.

6.2.2 **FEM→GA** By this different domain division strategy the gene evaluation value $p_i(t).stress[k]$ must be prepared before SX operation. Discussion in section 6.1 demonstrates element stress ranking number can determine the general boundary profile and the detail differences of elments have influences on the inner structure of the final solution. According to this observation, **max-map** and **avg-map** are defined for FEM→GA to prepare $p_i(t).stress[k]$.

- **max-map**
  Gene evaluation value $p_i(t).stress[k]$ is defined by formula (8), where each gene corresponds to $M$ elements. For example, in Fig.16, the GA design domain is divided into 4 meshes and the FEM domain is divided into 16 meshes. $p_i(t).stress[1]$ is evaluated by the maximum of $element[1.1].stress$, $element[1.2].stress$, $element[1.3].stress$ and $element$ $[1.4].stress$.

$$p_i(t).stress[k] = max.\{element[k.m].stress \\ |m = 1 \ldots M\} \quad (8)$$

- **avg-map**
  For this method, each gene is defined as the average of the corresponding FEM elements stress, which can be stated by formula (9). For this method, both the element stress ranking number and influences of neighboring elements are considered.

$$p_i(t).stress[k] = (\sum_{m=1}^{M} element[k.m].stress)/M \quad (9)$$

6.2.3 **Experiment** The experiment problem is same with section 6.1. The GA domain and FEM domain are divided into 30 × 10 and 120 × 40 meshes, respectively. The resulting topology by max-map and avg-map method are shown in Fig.18 and Fig.19.

Fig.18 Resulting Topology by max-map Method

Fig.19 Resulting Topology by avg-map Method

Comparison of Fig.18 and Fig.19 shows the resulting topology by max-map has checkerboard-like phenomena but the topology by avg-map is much simple. These results further verify the differences of elements is the key for suppressing checkerboard-like phenomena. In addition, the results also show it is practical to use different domain division strategy for GA and FEM to reducing GA design domain and increasing the accuracy of FEM.

## 7. Conclusion

In our previous study, a SX was introduced to solve structural topology optimization problems. In this paper, three types of SX parameter are discussed. First, generation alternation models are used to speed up convergence. Comparison of the SGA model and ER model showed that the latter can quickly search a more optimal solution. Second, four different meshes are adopted to discuss the mesh dependency of SX through a number of experiments for an MBB beam problem. The experiments yielded certain square mesh size is sufficient for SX to do structural topology optimization. Finally, a stress ranking method was used to discuss the impact of element stress on the final topology. Experiments demonstrated that element stress rank number can determine the general outline profile but with serious checkerboard-like phenomena in inner structure. To reduce GA design variables and obtain a good design a different domain division strategy was used for GA and FEM. Max-map and avg-map methods are introduced to map several elements of FEM to one gene of GA. Experiment results showed avg-map was practical for obtainning a simple topology with small GA search space. In addition, results comparison further verified differences of elements is the key for suppressing checkerboard-like phenomena.

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