Discussion of the crossover method of interactive Genetic Algorithm for extracting multiple peaks on Kansei landscape

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***Abstract*—Interactive Genetic Algorithms (iGAs) are optimization techniques used to estimate customers' Kansei (Japanese term for computing that relates to human characteristics such as sensibility, perception, affection or subjectivity) because human subjective evaluations are replaced with the objective function of Genetic Algorithms (GAs). Applying iGAs to recommend a product to a customer is examined in our study. One of the requirements is to estimate multiple preferences of a user and reflect preferences in the recommended products shown to him or her. When users select their preferred products within a specific category, they might like various kinds of products. In our study, these preferences are defined as multimodal preferences. When searching products a user would want, the recommendation method displays the more favored products by considering this multimodal preference.**

Therefore, in this study, we discuss using an iGA to generate offspring by estimating and searching multiple peaks. Our proposed method estimates multiple peaks by clustering the parents that the customer has evaluated more favorably and generates the appropriate offspring by constructing the probabilistic model based on the distribution of parents within a cluster. We performed two experiments. In the first experiment, we confirmed that the participants of the experiment had multimodal preferences. In the second experiment, the participants operated one of two systems which implemented either the proposed method or conventional method. The comparison of results showed that the system that implemented the proposed method searched the participants' multimodal preferences more diversely than the system that implemented the conventional method.

I. Introduction

Interactive Genetic Algorithms (iGAs) is one of interactive evolutionary computations [1]. These methods optimize the objects based on a customer's Kansei by replacing a human subjective evaluation with the objective function of Genetic Algorithms (GAs) [2]. Fig. 1 shows the flow of the algorithm. At first, an iGA system user evaluates the displayed solutions according to their preferences. The iGA system selects the parents using the evaluation values and generates a new population by the crossover method. One generation comprises these steps. By repeating these steps for each generation, the displayed choices increasingly become more closely matched to the user's preferences. iGAs are used in the design problems that need human Kansei such as fashion design [3], hearing aid fittings [4], [5], and so on [6], [7].

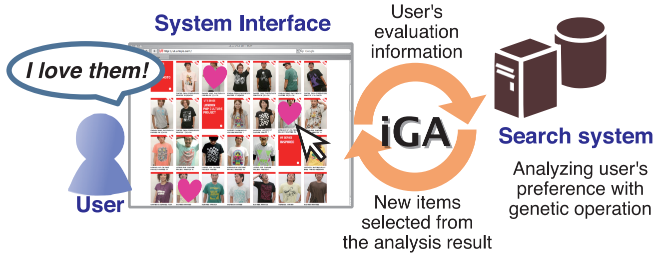


Fig. 1. Overview of iGA system

In our study, applying an iGA system to product recommendations on an online shopping site is examined. Online shopping sites need to effectively display the products that a customer really wants to buy because many sites offer a multitude of products but can only display a small number of them at one time. When customers want to buy something but cannot understand their needs by themselves, a site that uses an iGA as the product recommendation algorithm can display the products that most closely match their preferences. One of the problems in performing iGA is how to make product recommendations that take into account the customers' multiple preferences [8], [9]. A customer might like various products that belong to the same category. For example, in the case of determining T-shirt color preferences, a user might like not only white but also blue. Therefore, the product recommendation needs to reflect the customers' multiple preferences (multimodal preference) instead of focusing products based on one of preferences. However, conventional iGAs were designed to search for an optimized solution. This means that an iGA is not adequate to search for each of the multiple peaks. In this study, we discuss a crossover method that estimates multimodal preferences using a clustering method and searches for each peak using principal component analysis (PCA).

II. Crossover Method for Searching Multiple Peaks

A. Problems

We assume that human Kansei is modeled as a function that uses parameters of the object as inputs and the degree

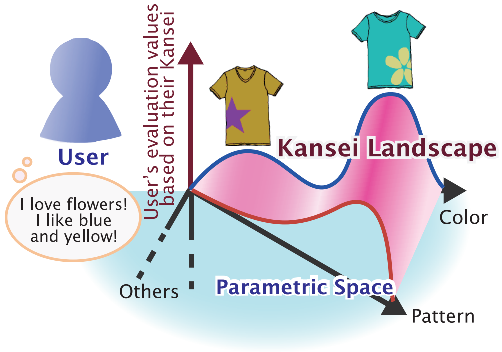


Fig. 2. Kansei landscape

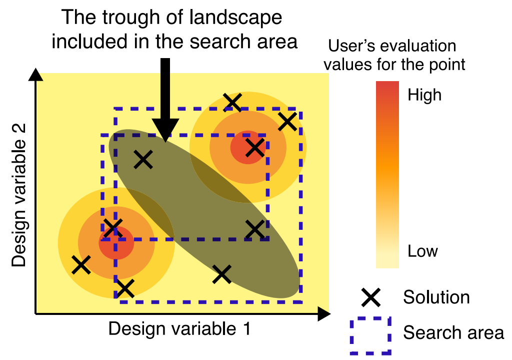


Fig. 3. Search process using the conventional method with a landscape having multiple

of matching to a customer's Kansei as outputs. Fig. 2 shows an example of the function. The parametric space treats the design variables such as T-shirt color and pattern as the axes. Each point of the parametric space shows one design. The hypersurface that maps the evaluation values of the customer's Kansei as the height (y axis) is called the Kansei landscape [10], which is the model of human Kansei used in our study. The product recommendation algorithms should determine the top of peaks on the Kansei landscapes. Therefore, we examined an iGA that interactively searches the tops of peaks.

In some cases, the Kansei landscape has multiple peaks that are almost of the same height when the user selects the products because the user might have multiple preferences within the same product category. However, conventional iGAs are designed to search for a single optimized solution. Fig. 3 shows an example. When there are multiple peaks in the Kansei landscape that are almost of the same height, the conventional methods focus the search area on only one peak searching not only the peaks but also the low evaluation area between them. In the case of such a multimodal landscape, we thought that the iGA's performance could be improved by extracting multiple peaks and searching them individually.

Furthermore, the presentation of multiple preferences increases the user's satisfaction. Conventional iGAs are not able to detect multiple preferences during one operation because they converge the population to a single peak, and if they are repeated, they cannot always find all multiple peaks. In addition, repeating the evaluation from the initial state is impossible in the domain of product recommendations. Therefore, it is necessary to search several peaks at a time.

B. Proposed Method

In this study, we discuss a crossover method that estimates the multiple peaks and searches their tops using a clustering method and PCA.

1) Estimation of the multiple peaks by the clustering method: First, to estimate the location of multiple peaks, we use the clustering method. When the clustering method is used on the design variable space, the parents selected from the population get divided into several dense groups as clusters. Each cluster is considered as a candidate location for a preference peak. Incidentally, the number of clusters is determined automatically according to the parents' distribution because the number of a user's preference peaks was unknown beforehand. For example, there are clustering methods that have a function to determine the best number of clusters [11], [12]. Alternatively, indicators of the accuracy of the clustering result such as gap statistics [13], [14] or silhouette statistics [15], [16] are used. These indicators are obtained from the results of inputting the various numbers of clusters into clustering methods, which used these numbers to select the proper number of clusters based on the indicators.

In the experiment described in IV, we used k-means as the clustering method and silhouette statistics as the indicator. The details are described in IV-C1.

2) Searching for solutions within the peaks using principal component analysis: Next, to search within each peak, a probabilistic model is constructed on the basis of the parents' distribution within a peak. The proposed method constructs the multidimensional normal distribution from the positions of the parent solutions that belong to a cluster and generated the children using a normal random number based on the distribution. Then, PCA [17] is conducted to embed the correlations among design variables into the distribution [18], [19].

When the number of design variables is n and a cluster has m parents, they are represented as an $m{*}n$ matrix X. Then, the matrix T is obtained by translating X so that the mean of each of the design variables becomes zero. We refer to the variance-covariance matrix of T as the matrix S. The eigenvalues and eigenvectors of matrix S are obtained by conducting PCA, and the rotation matrix A is constructed by ranking the eigenvectors in a column in the descending order of the absolute values of eigenvalues. The parents are mapped into the space where they have no correlation among dimensions by multiplying the rotation matrix A to the translated parents matrix T.

$$Y = TA \tag{1}$$

The multidimensional normal distribution is constructed from Y, and then, the children solutions $Y_{offspring}$ are generated. When children are generated, their positions are converged near the origin of space if the raw distribution is used. To avoid excessive convergence, each variance of axis of the multidimensional normal distribution is multiplied by α and the offspring are generated in a wider space. By multiplying $Y_{offspring}$ by A^{-1} that is the inverse matrix of A, $Y_{offspring}$ are mapped into the original space from the space with a basis matrix of A.

$$X_{offspring} = Y_{offspring} A^{-1} \tag{2}$$

The mean vector of X are added to $X_{offspring}$, and the offspring are added to the next generation population. By applying these operations to all clusters, the next generation population is generated. Incidentally, it is necessary to define the number of offspring in each cluster so that the sum is coincident with the population size. In the implementation shown in IV-C, the number of offsprings in each cluster is determined on the basis of the number of parents that belong to a cluster.

In this study, we performed two experiments to confirm the effectiveness of the proposed method. The first experiment verified the existence of the multimodal preference in the Kansei landscape. Then, on the basis of this result, the second experiment confirmed that the proposed method was able to search multiple peaks.

III. Experiment for Confirming Multiple Peaks on Kansei Landscape

A. Outline

To verify that a user has a multimodal preference, we obtained the experiment participants' approximate Kansei landscapes for each of three applications. The experiment participants comprised eight males and four females in their 20s. In the experiment, each design variable was divided evenly by the grids, and the grid points were treated as the sample points. The participants evaluated how much they liked these samples. The evaluation values of the samples were used to estimate an approximate landscape, and we determined the number of peaks that were present.

B. Experimental System

Three patterns were used as the experimental applications. Table I shows these design variables. To easily visualize the results, only two design variables, i.e., color and size were used to make minute changes. Colors were selected from a color space based on hue, saturation, and brightness (HSB) [20]. Saturation and brightness values were kept static, and only hue was used as the design variable. The radius of dot was defined by multiplying the distance between neighboring dots by the parameter "Rate of dot size". Fig. 4 shows examples of the solutions that were shown to the participants.

Fig. 5 shows the experimental interface. Each of design variables was divided into 10 points. We created 100 unique solutions by combining each point of one variable with each point of the other variable and displayed them on the experimental interface. Under each solution, there are seven radio buttons and participants chose the level of button to evaluate how much they liked each solution.

TABLE I
Design Variables

Application	Design variable 1	Design variable 2
Dotted	Hue of dot	Rate of dot size
Arabesque	Hue of front color	Hue of background color
Plaid	Hue of first color	Hue of second color

(a) Dotted (The number of dot was static and the size of dot was parameterized.)

(b) Arabesque (The color of arabesque pattern was treated as front color.)

(c) Plaid (Two colors except white were parameterized.)

Fig. 4. Examples of displayed solutions

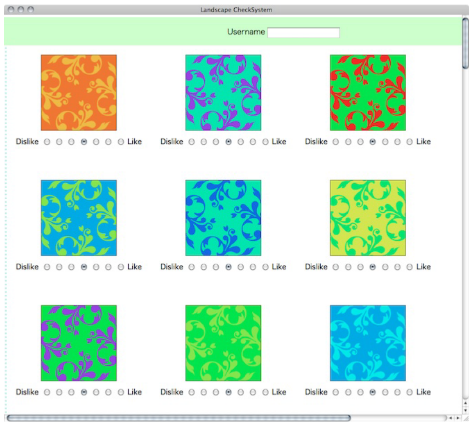


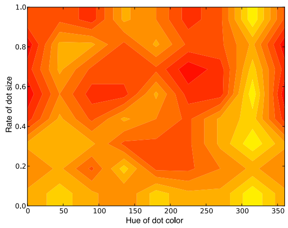
Fig. 5. Experimental interface for confirming multiple peaks in the Kansei landscape

C. Experimental Procedure

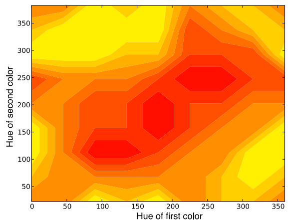
If a participant's evaluation criterion changes during a trial run, it is difficult to obtain accurate landscapes. To address this problem, we selected the participants' preferences of upholstery patterns for furniture as the evaluation criterion because the fluctuation in those preferences was thought to be small. The participants were instructed to evaluate how much they preferred each solution as a fabric pattern that was used for a curtain, bed cover, and sofa cover when renovating a room. The participants operated three trial runs using Dotted, Arabesque or Plaid. The operation order of trial runs was counterbalanced among participants.

D. Result

Fig. 6 shows examples of landscapes obtained by the experiment. The axes are the design variables of the applications. The areas evaluated as the most preferred are colored red, and those evaluated as the least preferred are colored yellow. The



(a) Landscape with multiple peaks (Application: Dotted)



(b) Landscape with a single peak (Application: Plaid)

Fig. 6. Examples of the estimated Kansei landscapes

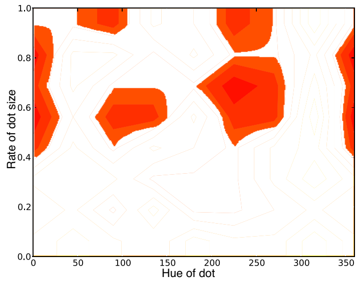


Fig. 7. Multimodal landscape shown in Fig. 6(a) showing only the peaks

areas that the participants did not evaluate are interpolated by the bilinear interpolation method using the evaluation values of the sample points. Fig. 6(a) is a landscape that has multiple peaks, and Fig. 6(b) is a landscape that has a single peak.

To define the peaks within a landscape, we made a frequency histogram from the evaluation values and calculated the threshold value dividing the upper 25% of each landscape. Any area that has an evaluation value higher than the threshold value is defined as a preference peak. Fig. 7 is the landscape image created by coloring all areas except the peaks in Fig. 6(a) white.

We examined the multiple preferences of the participants' Kansei by counting the number of these peaks in all landscapes. Fig. 8 shows a graph of the frequency of the number of peaks. The horizontal axis is the number of peaks and the vertical axis is the number of landscapes for each peak number. This graph includes the results for all three applications. In the graph, 27 of 36 landscapes have more than 2 peaks. This result proved that the majority of the Kansei landscapes were multimodal. Therefore, it is thought that a method is needed to search the landscapes for multiple peaks.

IV. Experiment for Verifying the Effectiveness of the Proposed Crossover Method

A. Outline

This experiment aimed at confirming that the proposed method was able to estimate the multiple peaks and search for better solutions within a peak. The experiment participants operated the following two systems to compare their results. The proposed system implements the proposed method as a crossover method and conventional system implements BLX-α [21] as a conventional crossover method. We analyzed the

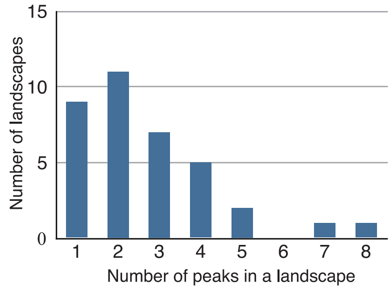


Fig. 8. Frequency graph of the number of landscapes for every number of peaks

Fig. 9. Experimental interface for the analysis of the proposed method

search results of both systems using the approximate landscapes obtained in the first experiment (hereafter referred to as the preliminary experiment) and focusing on the following items.

1) The improvement of the evaluation values along with the change of generations.
2) The performance of searching for multiple peaks.

The participants were the same as the preliminary experiment, and they used the same three experimental applications.

B. Experimental Interface

Fig. 9 shows the experimental interface. The system displayed 25 solutions, and the participants chose their favorites by clicking these images. When the user clicked on an image, the frame color of the image changed to red. After evaluating all displayed solutions, the participant clicked the Next Page button at the bottom of the interface, and 25 new solutions were displayed. Each sequence of these evaluations is called a page. The participants evaluated the solutions repeatedly until a pop up was displayed that notified the participant of the end of the trial.

C. Experimental System

Table II represents the parameters that are shared in the proposed and conventional methods. Both systems have the same parameters, except for their crossover methods. The following are the details of the implementation of the proposed, conventional, and selection methods.

TABLE II
EXPERIMENTAL PARAMETERS

Atribute	Value
Population Size	25
Generation Size	12
Selection Size	13
Crossover Rate	1.0
Mutation Rate	0.2
Mutation Method	Uniform mutation

1) The proposed method: The proposed system used k-means as the clustering method. The k-means calculation needs the prepared number of clusters. However, as described in II-B1, the proper number of peaks, i.e., the proper number of clusters is unknown because a user's landscape has not been obtained yet when the user uses an iGA system. Therefore, silhouette statistics [16] were introduced to automatically determine the number of clusters. When the sum of the distances between solutions within the same cluster is smaller and the sum of the distances between solutions that belong to a neighbor cluster is bigger, the statistics are bigger. The proposed system changes the number of clusters that is given to the k-means method from 2 to 8. Then, the number when the silhouette statistics were the largest was adopted.

In order to avoid excessive convergence of offspring as mentioned in II-B2, the each dimensional standard variance of the normal distribution were magnified 1.4 times.

2) The conventional method: The conventional system randomly selects two of the selected parents and applies the BLX-α process to them repeatedly, using an α of 0.2.

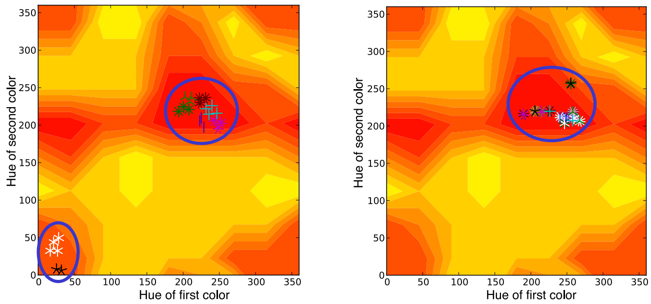
3) The selection method: The selection method selected the parents from the solutions that were chosen by the participant. This means the number of selected solutions might not meet the number needed by the crossover method in the earlier pages. In this experiment, the minimum number of parents for the crossover method was defined as 13, which was half of the population size. Until the defined number of selected solutions was stored, the random solutions were generated, and the generation was not changed. After the second generation, until the number of parents became greater than 13, the system selected the solutions that were clicked on the latest and newer pages.

D. Experimental Procedure

The participants were instructed to select the fabric patterns that they would prefer to use as upholstery on furniture or as the cloth for a curtain, bed cover, and sofa cover when renovating their rooms. Each participant performed the six trial runs, combining each of the two systems and the three applications, such as Dotted, Arabesque, and Plaid. We varied the order of the applications and that of the two systems for each of the participants.

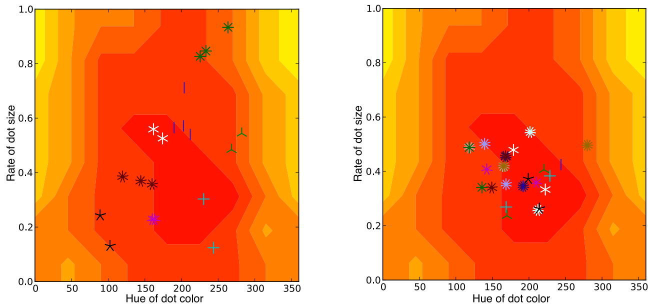
E. Result

1) Discussion of the search results: Fig. 10 and Fig. 11 show examples of the search results. The axes are the design variables of the applications. The points are the selected



(a) Search result by the proposed system
(b) Search result by the conventional system

Fig. 10. Search results in the last generation of a participant generating a multimodal landscape for the Plaid



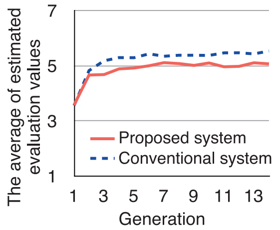
(a) Search result using the proposed system
(b) Search result using conventional system

Fig. 11. Search results in the last generation of a participant who generated a unimodal landscape for Dotted

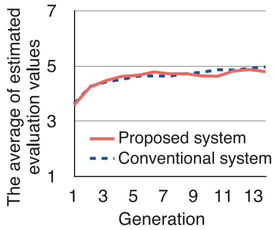
parents in the last generation. In particular, in Fig. 10(a) and Fig. 11(a) that show the search results using the proposed system, the points that have the same color belong to the same cluster. The landscapes behind the points were obtained in the preliminary experiment.

Fig. 10 represents one participant's search results in the last generation when the application was the Plaid. There are two peaks on this landscape because each design variable is joined end-to-end. While the conventional system searched only one peak, the proposed system was able to search two peaks using different clusters separately, as shown by the circle in Fig. 10(a).

Fig. 11 shows the search results of a participant who generated a unimodal landscape for the Dotted. The solutions searched by the conventional system were converged at the top of a peak, whereas those searched by the proposed system were widely halfway up a peak. This means that the performance of the proposed method was worse compared to that of the conventional method. Fig. 11(a) demonstrates that small, multiple areas were searched by the proposed system because the parents were divided into too many clusters. The excessive classification is considered the reason that the proposed system distributed the offspring broadly and failed the convergence at the top of the peak. The performance of the proposed method was affected heavily by the algorithm that automatically determined the number of clusters. Therefore,

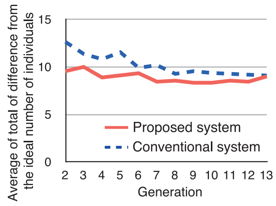


(a) Results from landscapes with a single peak (average of the 9 trials)

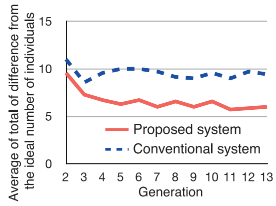


(b) Results from landscapes with multiple peaks (average of 27 trial runs)

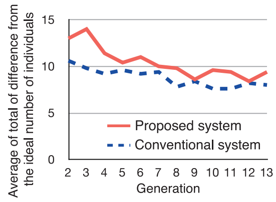
Fig. 12. Averages of evaluation values estimated for each generation from the landscapes obtained in the preliminary experiment



(a) The average of 11 landscapes with 2 peaks



(b) The average of 7 landscapes with 3 peaks



(c) The average of 5 landscapes with 4 peaks

Fig. 13. Generational average of sums of differences of each peak between the ideal size and actual generation size

we need to study the benchmark of setting the number of clusters, including the silhouette method, both of which were used in this experiment.

2) Improvement of the evaluation values with each change of generation: To confirm that the proposed method searches better with each change of generation, we examined whether the generational average of the evaluation values progressively increased. However, the detailed evaluation values could not be obtained in this experiment because the evaluation performed on the experimental interface was binary. To examine the transition of the evaluation values in detail, we estimated an evaluation value of the solutions from the outline of the landscapes obtained in the preliminary experiment.

Fig. 12 shows a graph of the generational average of the estimated evaluation values. Fig. 12(a) shows the average of the unimodal landscapes, and Fig. 12(b) shows the average of the multimodal landscapes. The horizontal axis indicates the number of generations, and the vertical axis indicates the average of the estimated evaluation values. The solid red line represents the results of the proposed method and the dotted blue line represents those of the conventional method.

Fig. 12(a) shows that the conventional method garnered higher averages than the proposed method on the unimodal landscapes. On the other hand, the averages of both methods on the multimodal landscapes are nearly equal, as shown in Fig. 12(b). These results show that the search performance of both methods are almost the same if a landscape has multiple peaks. However, the conventional method searches more effectively when a user has a unimodal landscape.

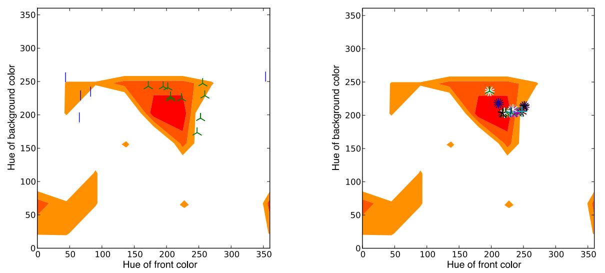
3) Performance of searching for multiple peaks: To confirm that the proposed method searched multiple peaks, we measured the differences between the number of solutions expected to be generated in a peak (the ideal size) and the number actually generated in the peak. The ideal size is defined as follows. First, the area of each peak obtained in the preliminary experiment is measured. Then, the ratio of the area to the total peak area is calculated, and the ratio is multiplied by the population size. This value is treated as the ideal size for the peak.

Fig. 13 shows the generational average of the sums for each peak of the differences between the ideal size and actual generation size for landscapes having 2, 3, and 4 peaks. The horizontal axis represents the number of generations, and the vertical axis represents the averages of the sums of the differences. The smaller the averages were, the more the diverse search was thought to be effective. We did not graph the averages of the landscapes with more than 5 peaks because there were less than 5 landscapes for each number of peaks. The graph shows the averages of the proposed system are small in the landscapes having 2 or 3 peaks. On the other hand, the averages of the conventional system are small in the landscapes with 4 peaks. According to the results shown in III-D, the participants who generated landscapes with two or three peaks made up half of all the participants. Therefore, because the difference of the proposed method is smaller and better in the results of these participants, the proposed method is considered useful.

To determine the reason for the conventional method to be superior for landscapes with 4 peaks, we examined the search results from the participant who generated the 4 peak landscape, as shown in Fig. 14. The central area of this landscape is treated as the widest peak. Naturally, the ideal size assigned to this peak is large. The conventional system searched only the right side of this area for most solutions. However, the proposed system searched both the right and left sides separately using the other cluster. Therefore, the proposed system searched more widely and tended to suggest many solutions that were out of the defined peak area. As a result, the averages of the differences of ideal sizes of the proposed system were bigger than those of the the conventional system. In fact, Fig. 14 indicates that the proposed system searched the domain of the peaks correctly and displayed a diverse selection of items.

F. Summary of the experimental results

In this experiment, we verified the effectiveness of the proposed method by examining (1) the improvement of the evaluation values with each change of the generation and (2)



(a) Search result using the proposed method

(b) Search result using the conventional method

Fig. 14. Examples of search results on the landscape having 4 peaks

the performance of searching for multiple peaks. In terms of the improvement of the estimated evaluation values, the conventional method was superior at the case of unimodal landscapes. However, for multimodal landscapes, both methods increased the evaluation values in almost the same way. The analysis of how each method searched multiple peaks showed that the proposed method searched more peaks than the conventional method. These results showed that the proposed method displays a more diverse selection of items and reflects the user's multiple preferences by searching peak-by-peak.

V. Conclusion

To apply iGA to make product recommendations, we proposed a crossover method that searches according to multiple preferences. The proposed method estimates the multiple peaks in a user's Kansei landscape using the clustering method and searches the top of each peak using PCA. Two experiments were performed for this study. The first experiment proved that multiple preferences exist by extracting multiple peaks from the landscapes that the participants generated on each of the three applications. The second experiment showed the effectiveness of the proposed method in contrast with the conventional method using BLX-α. The search results showed that the proposed method searches more peaks than the conventional method. However, the proposed method tends to estimate too many peaks when a landscape has a single wide peak. This tendency was also shown on the transition of the evaluation values. For the unimodal landscapes, the averages of the estimated evaluation values of the conventional method were higher than those of the proposed method. Conversely, with the examination of the diversity of the performance, the proposed method searched more peaks than the conventional method and covered the area of the peaks with the solutions. In the future, we need to study the technique that determines the number of clusters because it affected the accuracy of the search process significantly, as shown in the second experiment. Also, only two design variables were used for each application discussed in this study to visualize the experimental results more easily. Therefore, it is necessary to examine the performance of the proposed method when the number of design variables is increased.

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